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**WIENER PACK: A SUBROUTINE PACKAGE FOR
COMPUTING PROBABILITIES ASSOCIATED WITH
WIENER AND BROWNIAN BRIDGE PROCESSES**

C.F. Chung

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WIENER PACK: A SUBROUTINE PACKAGE FOR COMPUTING PROBABILITIES ASSOCIATED WITH WIENER AND BROWNIAN BRIDGE PROCESSES

Abstract

A subroutine package *WIENER PACK* written in *FORTRAN-77* has been developed to compute the probabilities associated with Wiener and Brownian bridge processes of the following forms:

$$\begin{aligned} P \{ W(t) \leq \gamma + \delta t : a \leq t \leq b \}; \\ P \{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \}; \\ P \{ B(t) \leq \gamma : a \leq t \leq b \}; \\ P \{ |B(t)| \leq \gamma : a \leq t \leq b \}, \end{aligned}$$

where $W(\cdot)$ and $B(\cdot)$ are Wiener and Brownian bridge processes, respectively.

Résumé

Un ensemble de sous-routiers, *WIENER PACK*, en *FORTRAN-77*, a été mis au point pour calculer les probabilités associées aux processus de liaison Wiener et Brown dont la symbolisation est la suivante:

$$\begin{aligned} P \{ W(t) \leq \gamma + \delta t : a \leq t \leq b \}; \\ P \{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \}; \\ P \{ B(t) \leq \gamma : a \leq t \leq b \}; \\ P \{ |B(t)| \leq \gamma : a \leq t \leq b \}, \end{aligned}$$

Les symboles $W(t)$ et $B(z)$ identifient les processus de liaison Wiener et Brown.

INTRODUCTION

Weak convergence and strong approximation of empirical, empirical quantile, product-limit and Quantile-Quantile processes to Brownian bridge and Wiener processes are well known (Breslow and Crowley, 1974; Csörgö and Révész, 1978; Csörgö and Horváth, 1985; Aly et al., 1985; Aly and Bleuer, 1983). These theories can, in principle, be used to construct confidence bands for their statistical functions.

When these methodologies are applied to statistical problems, however, the computation of probabilities for functionals of interest for the approximating Gaussian processes plays a crucial role, and the required computations on functionals of appropriate Gaussian processes are frequently complicated. Without such computations, recent statistical advancements using weak convergence and strong approximation methodologies cannot be used in practice.

A subroutine package *WIENER PACK* written in *FORTRAN-77* has been developed to compute the probabilities associated with Wiener and Brownian bridge processes of the following forms:

- (A.1) $P\{ W(t) \leq \gamma : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0;$
- (A.2) $P\{ |W(t)| \leq \gamma : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0;$
- (A.3) $P\{ W(t) \leq \gamma + \delta t : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0, \gamma + \delta b \geq 0;$
- (A.4) $P\{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0, \gamma + \delta b \geq 0;$
- (A.5) $P\{ B(t) \leq \gamma : a \leq t \leq b \}, a \geq 0, b \leq 1, \gamma > 0;$
- (A.6) $P\{ |B(t)| \leq \gamma : a \leq t \leq b \}, a \geq 0, b \leq 1, \gamma > 0.$

where $W(\cdot)$ and $B(\cdot)$ are Wiener and Brownian bridge processes, respectively.

In addition, for given probability p , the package enables the user to compute any one of the parameters a , b or γ in the formulae (A.1), (A.2), (A.5), and (A.6), by specifying the other two parameters.

Some special cases of the probabilities of (A.5) and (A.6) have been developed by Csörgö and Horváth (1981), Barr and Davidson (1973), Koziol and Byar (1975), and Schumacher (1984) among many others. A computer program package called WIENER PACK has been developed to compute the probabilities of the above forms for any given values of the parameters a , b , γ and δ . Inversely, the package also enables one to compute any one of the parameters a , b and γ in (A.1), (A.2), (A.5) and (A.6) for a given probability level and given values for the other two of the parameters a , b and c .

Obviously, (A.1) and (A.2) are special cases of (A.3) and (A.4), respectively. It can also be shown that (A.5) and (A.6) are again special cases of (A.3) and (A.4), respectively. However, in actual computations, each of the formulae related to Wiener processes shown is programmed separately for computational efficiency.

In this paper, some of the available methods for computing probabilities associated with Wiener and Brownian bridge processes are summarized. More detailed discussions and proofs of the formulae quoted here are given in Chung (1986).

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I would like to thank Dr. M. Csörgö, Carleton University for his valuable remarks and suggestions.

THEORETICAL BACKGROUND FOR WIENER PROCESS

Definition of Wiener process

A stochastic process $\{W(t) : 0 \leq t < \infty\}$ is called a Wiener process if :

- (i) $W(t) - W(s) \sim N(0, t-s)$ for all $0 \leq s \leq t < \infty$ and $W(0) = 0$;
- (ii) $W(t)$ is an independent increment process, that is $W(t_2) - W(t_1)$, $W(t_4) - W(t_3)$, ..., $W(t_{2i}) - W(t_{2i-1})$ are independent random variables for all $0 \leq t_1 < t_2 < \dots < t_{2i-1} < t_{2i} < \infty$ ($i = 2, 3, \dots$), and
- (iii) the sample path function $W(t)$ is continuous in t with probability 1.

Note that (i) and (ii) imply that the covariance function of a Wiener process is

$$(1) \quad E(W(s)W(t)) = s \wedge t.$$

Conversely, a continuous Gaussian process having the covariance function of (1) is a Wiener process.

Formula for $p = P\{W(t) \leq \gamma + \delta t : 0 \leq t \leq b\}$.

Case 1.

Let $b > 0$, $\gamma > 0$ and $\gamma + \delta b \geq 0$. Then

$$(2) \quad P\{W(t) \leq \gamma + \delta t : 0 \leq t \leq b\} = \Phi[(\gamma + \delta b)/\sqrt{b}] - e^{-2\gamma\delta} \Phi[(-\gamma + \delta b)/\sqrt{b}],$$

where

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{1}{2}h^2} dh.$$

When $\delta = 0$, then

$$(3) \quad P\{W(t) \leq \gamma : 0 \leq t \leq b\} = 2\Phi(\gamma/\sqrt{b}) - 1.$$

Case 2.

If $\gamma > 0$ and $b = 0$, then $p = 1$.

Formula for $p = P\{W(t) \leq \gamma + \delta t : a \leq t \leq b\}$

Case 1.

Let $a > 0$, $\gamma > 0$, $a < b$, $\gamma + \delta a \geq 0$ and $\gamma + \delta b \geq 0$.

Then we have

$$(4) \quad P\{W(t) \leq \gamma + \delta t : a \leq t \leq b\} \\ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\gamma + \delta a)/\sqrt{a}} e^{-1/2x^2} \left[\Phi \left[(\gamma + \delta b - \sqrt{ax})/\sqrt{b-a} \right] - e^{-2(\gamma + \delta a - \sqrt{ax})\delta} \Phi \left[(-\gamma - 2\delta a + b\delta + \sqrt{ax})/\sqrt{b-a} \right] \right] dx$$

Note that when $\delta = 0$, we have

$$(5) \quad P\{W(t) \leq \gamma : a \leq t \leq b\} \\ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\gamma/\sqrt{a}} e^{-1/2x^2} \left[2 \Phi \left[(\gamma - \sqrt{ax})/\sqrt{b-a} \right] - 1 \right] dx.$$

Rényi (1953, (3.6) on page 208) and Csörgő (1967, (3.4) on page 554) have also given (5).

Case 2.

Let $a > 0$, $\gamma > 0$, $a = b$ and $\gamma + \delta a \geq 0$. Then,

$$(6) \quad P\{W(a) \leq \gamma + \delta a\} = \Phi \left[(\gamma + \delta a)/\sqrt{a} \right].$$

Formula for $P\{\gamma_2 + \delta_2 t \leq W(t) \leq \gamma_1 + \delta_1 t : 0 \leq t \leq b\}$.

Case 1.

(Anderson, 1960, (4.32) on page 180). Let $\gamma_1 > 0$, $\gamma_2 < 0$, $b > 0$, $\gamma_2 + \delta_2 b \leq \gamma_1 + \delta_1 b$. Then we have

$$(7) \quad P\{\gamma_2 + \delta_2 t \leq W(t) \leq \gamma_1 + \delta_1 t : 0 \leq t \leq b\} \\ = 1 - \Phi(w/\sqrt{b}) - \Phi(-v/\sqrt{b}) \\ + \sum_{k=1}^{\infty} \left[e^{s_1} [\Phi(q_1) - \Phi(p_1)] + e^{s_2} [\Phi(p_2) - \Phi(q_2)] \right. \\ \left. - e^{s_3} [\Phi(q_3) - \Phi(p_3)] - e^{s_4} [\Phi(p_4) - \Phi(q_4)] \right]$$

where

$$\begin{aligned} \gamma' &= \gamma_1 - \gamma_2, \quad \delta' = \delta_1 - \delta_2, \quad v = \gamma_1 + \delta_1 b, \quad w = \gamma_2 + \delta_2 b, \\ s_1 &= -2 [k^2 \gamma' \delta' + k(\gamma_1 \delta_2 - \gamma_2 \delta_1)], \\ s_2 &= -2 [k^2 \gamma' \delta' - k(\gamma_1 \delta_2 - \gamma_2 \delta_1)], \\ s_3 &= -2 [k^2 \gamma' \delta' + k(\gamma' \delta_2 + \delta' \gamma_2) + \gamma_2 \delta_2], \\ s_4 &= -2 [k^2 \gamma' \delta' - k(\gamma' \delta_1 + \delta' \gamma_1) + \gamma_1 \delta_1], \\ p_1 &= (-2k\gamma' + w)/\sqrt{b}, \quad q_1 = (-2k\gamma' + v)/\sqrt{b}, \\ p_2 &= (-2k\gamma' - w)/\sqrt{b}, \quad q_2 = (-2k\gamma' - v)/\sqrt{b}, \\ p_3 &= (-2k\gamma' + w - 2\gamma_2)/\sqrt{b}, \quad q_3 = (-2k\gamma' + v - 2\gamma_2)/\sqrt{b}, \\ p_4 &= (-2k\gamma' - w + 2\gamma_1)/\sqrt{b}, \quad q_4 = (-2k\gamma' - v + 2\gamma_1)/\sqrt{b}. \end{aligned}$$

From (7), it can be derived that

From (7), it can be derived that

$$(8) \quad P\{ \gamma_2 + \delta_2 t \leq W(t) \leq \gamma_1 + \delta_1 t : 0 \leq t \leq b \} \\ = P\{ -\gamma_1 - \delta_1 t \leq W(t) \leq -\gamma_2 - \delta_2 t : 0 \leq t \leq b \}.$$

Note that when $\gamma_1 = -\gamma_2 = \gamma (>0)$ and $\delta_1 = -\delta_2 = \delta (\geq 0)$, then

$$(9) \quad P\{ |W(t)| \leq \gamma + \delta t : 0 \leq t \leq b \} \\ = 1 - 2 \Phi(-u/\sqrt{b}) - 2 \sum_{k=1}^{\infty} (-1)^{k+1} \\ e^{-2k^2\gamma\delta} \left[\Phi[(-2k\gamma+u)/\sqrt{b}] - \Phi[(-2k\gamma-u)/\sqrt{b}] \right]$$

where $u = \gamma + \delta b$. Gillaspie and Fisher (1979, $P(\gamma, \delta, b)$ on page 921) have also discussed (9).

When $\delta_1 = \delta_2 = 0$ in (7), we have

$$(10) \quad P\{ \gamma_2 \leq W(t) \leq \gamma_1 : 0 \leq t \leq b \} \\ = 1 - 2 \sum_{k=1}^{\infty} (-1)^{k+1} \left[\Phi[(-k\gamma' + \gamma_1)/\sqrt{b}] + \Phi[(-k\gamma' - \gamma_2)/\sqrt{b}] \right]$$

where $\gamma' = \gamma_1 - \gamma_2$. It was also denoted by Feller (1966, (5.8) on page 329) as $\lambda. (b, \gamma_1)$. In addition Feller (1966, (5.9) on p. 330) and M. Csörgö (1967, (2.5) on page 553) have shown an alternative formula for (10) as follows:

$$(11) \quad P\{ \gamma_2 \leq W(t) \leq \gamma_1 : 0 \leq t \leq b \} \\ = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k+1} \exp \left[\frac{-(2k+1)^2 \pi^2 b}{2(\gamma_1 - \gamma_2)^2} \right] \sin \left[\frac{(2k+1) \pi \gamma_1}{\gamma_1 - \gamma_2} \right].$$

Feller (1966) noted that the series in (10) converges quickly when b is small whereas in (11) it converges rapidly when b is large.

When $\gamma_1 = -\gamma_2 = \gamma$, (10) and (11) are reduced to

$$(12) \quad P\{ |W(t)| \leq \gamma : 0 \leq t \leq b \} \\ = 1 - 4 \sum_{k=1}^{\infty} (-1)^{k+1} \Phi[-(2k-1)\gamma/\sqrt{b}] \\ = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \exp \left[\frac{-(2k+1)^2 \pi^2 b}{8\gamma^2} \right].$$

When $b = 1$ in (12), Feller (1966, (5.11) on page 330) has denoted it as $L(\gamma)$.

Case 2.

Let $\gamma_1 > 0$, $\gamma_2 < 0$, $b = 0$. Then the probability is simply 1.

Formula for $P\{ \gamma_2 + \delta_2 t \leq W(t) \leq \gamma_1 + \delta_1 t : a \leq t \leq b \}$.

Case 1.

Let $\gamma_1 > 0$, $\gamma_2 < 0$, $a > 0$, $\gamma_2 + \delta_2 b \leq \gamma_1 + \delta_1 b$.

Then we have

$$(13) \quad P\{ \gamma_2 + \delta_2 t \leq W(t) \leq \gamma_1 + \delta_1 t : a \leq t \leq b \} \\ = \frac{1}{\sqrt{2\pi}} \int \frac{q'/\sqrt{a}}{q''/\sqrt{a}} \Gamma(q'' - \sqrt{ah}, \delta_2, q' - \sqrt{ah}, \delta_1, b-a) e^{-1/2h^2} dh,$$

where $q' = \gamma_1 + \delta_1 a$, $q'' = \gamma_2 + \delta_2 a$ and

$$(14) \quad \Gamma(u', v', u, v, w) \\ = P\{ u' + v't \leq W(t) \leq u + vt : 0 \leq t \leq w \},$$

which is discussed in (7).

When $\gamma_1 = -\gamma_2 = \gamma (> 0)$ and $\delta_1 = -\delta_2 = \delta (\geq 0)$, we have

$$(15) \quad P\{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \} \\ = \frac{2}{\sqrt{2\pi}} \int_0^{\gamma/\sqrt{a}} \frac{q/\sqrt{a}}{q - \sqrt{ah}, \delta, q - \sqrt{ah}, \delta, b-a} e^{-1/2h^2} dh,$$

where $q = \gamma + \delta a$. If $\delta = 0$ in (15), we obtain

$$(16) \quad P\{ |W(t)| \leq \gamma : a \leq t \leq b \} \\ = \frac{2}{\sqrt{2\pi}} \int_0^{\gamma/\sqrt{a}} \frac{\gamma/\sqrt{a}}{-\gamma - \sqrt{ah}, \gamma - \sqrt{ah}, b-a} e^{-1/2h^2} dh,$$

where

$$(17) \quad \Gamma'(u', u, w) = P\{ u' \leq W(t) \leq u : 0 \leq t \leq w \},$$

which is discussed in (10) and (11).

Case 2.

Let $\gamma_1 > 0$, $\gamma_2 < 0$, $a = b$ and $\gamma_2 + \delta_2 a \leq \gamma_1 + \delta_1 a$.

Then we have

$$(18) \quad P\{ \gamma_2 + \delta_2 a \leq W(a) \leq \gamma_1 + \delta_1 a \} \\ = \Phi[(\gamma_1 + \delta_1 b)/\sqrt{b}] - \Phi[(\gamma_2 + \delta_2 b)/\sqrt{b}].$$

THEORETICAL BACKGROUND FOR BROWNIAN BRIDGE PROCESS.

Definition of Brownian bridge process.

A stochastic process $\{B(t); 0 \leq t \leq 1\}$ is called a Brownian bridge if:

(i) the joint distribution of $B(t_1), \dots, B(t_n), 0 \leq t_1 \leq \dots \leq t_n \leq 1$, is Gaussian with $E(B(t)) = 0$; (ii) the covariance function is

$$(19) \quad EB(t)B(s) = s \wedge t - st; \text{ and}$$

(iii) the sample path function $B(t)$ is continuous in t with probability 1.

Note that (ii) implies that $B(0) = B(1) = 0$ almost surely. If $B(t) = W(t) - tW(1)$ ($0 \leq t \leq 1$) where $W(t)$ is a Wiener process, then $B(t)$ is a Brownian bridge. On the other hand, if $W(t) = (t+1)B[t/(t+1)]$, $t \geq 0$, then $W(t)$ is a Wiener process. Also $B(t) = (1-t)W[t/(1-t)]$, $0 \leq t < 1$ is a Brownian bridge process.

Formula for $p = P\{B(t) \leq c : 0 \leq t \leq b\}$.

Case 1.

Let $0 < b < 1$ and $c \geq 0$. Then

$$(20) \quad P\{B(t) \leq c : 0 \leq t \leq b\} \\ = \Phi[c/\sqrt{b(1-b)}] - e^{-2c^2} \Phi[-c(1-2b)/\sqrt{b(1-b)}].$$

Koziol and Byar (1975, (2.3) on page 508) have derived (20) denoting it as $G_b(c)$, although a misprint occurs in formula (2.3) on page 508 of their paper. i.e., $(T-T^2)^{-1/2}$ should be printed as $(T-T^2)^{-1/2}d$ in the probability argument of their formula (2.3). Hall and Wellner (1980, (2.8) on page 136) have also derived (20) denoting it as $G_b^+(c)$. However, their notation is confusing and their formula (2.8) is not correct.

Case 2.

Let $b = 1$. Then we have

$$(21) \quad P\{B(t) \leq c : 0 \leq t \leq 1\} = 1 - e^{-2c^2},$$

which is the limiting distribution function of the one-sided Kolmogorov-Smirnov statistic

$$\sqrt{n} \sup_{-\infty < x < \infty} (F_n(x) - F(x))$$

where $F_n(\cdot)$ is the empirical distribution function of a continuous distribution function $F(\cdot)$ (cf. Doob, 1949).

Formula for $p = P\{B(t) \leq c : a \leq t \leq b\}$.

Case 1.

Let $0 < a < b < 1$ and $c \geq 0$. Then

$$(22) \quad P\left\{\sup_{a \leq s \leq b} B(s) \leq c\right\} \\ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{c/(1-a)} e^{-1/2h^2} \left[\Phi\left[\frac{c\sqrt{1-a} - (1-b)\sqrt{ah}}{\sqrt{1-b}\sqrt{b-a}}\right] \right. \\ \left. - e^{-2c[c - \sqrt{a(1-a)}h]/(1-a)} \Phi\left[\frac{c(2b-a-1) + (1-b)\sqrt{a(1-a)}h}{\sqrt{1-a}\sqrt{1-b}\sqrt{b-a}}\right] \right] dh$$

Csáki (1981, (2.17) on page 257) has derived (22) as the limiting distribution function of a special empirical process of limited range and his formula has been quoted by Csörgö (1983, (4.2.10) on page 37).

Case 2.

Let $b=1$, $0 < a < b$, $c \geq 0$. Then, by the symmetricity property of the Brownian bridge process, we have

$$\begin{aligned} (23) \quad P\{B(t) \leq c : a \leq t \leq 1\} \\ = P\{B(t) \leq c : 0 \leq t \leq 1-a\} \\ = \Phi\left[\frac{c}{\sqrt{a(1-a)}}\right] - e^{-2c^2} \Phi\left[\frac{c(1-2a)}{\sqrt{a(1-a)}}\right]. \end{aligned}$$

Case 3.

Let $a = b$ ($\neq 0$ or 1) and $c \geq 0$. Then

$$(24) \quad P\{B(a) \leq c\} = \Phi\left[\frac{c}{\sqrt{a(1-a)}}\right].$$

Formula for $p = P\{|B(t)| \leq c : 0 \leq t \leq b\}$.

Case 1.

Let $b < 1$ and $c \geq 0$. Then we have

$$\begin{aligned} (25) \quad P\{|B(t)| \leq c : 0 \leq t \leq b\} \\ = 1 - 2 \Phi\left(-\frac{c}{\sqrt{b(1-b)}}\right) - 2 \sum_{k=1}^{\infty} (-1)^{k+1} e^{-2k^2 c^2} \\ \left[\Phi\left(\frac{-c(2k-2kb-1)}{\sqrt{b(1-b)}}\right) - \Phi\left(\frac{-c(2k-2kb+1)}{\sqrt{b(1-b)}}\right) \right]. \end{aligned}$$

Hall and Wellner (1980, (2.9) on page 136) have also shown (3.3.1) denoting it as $G_b(c)$.

Case 2.

Let $b = 1$ and $c \geq 0$. Then we have

$$\begin{aligned} (26) \quad P\{|B(t)| \leq c : 0 \leq t \leq 1\} \\ = 1 - 2 \sum_{k=1}^{\infty} (-1)^{k+1} e^{-2k^2 c^2}, \end{aligned}$$

which is the limiting distribution function of the two-sided Kolmogorov-Smirnov statistic or the empirical process

$$\sqrt{n} \sup_{-\infty < x < \infty} |F_n(x) - F(x)|$$

where $F_n(\cdot)$ is the empirical distribution function of a continuous distribution function $F(\cdot)$ (cf. Doob, 1949).

Formula for $p = P\{ |B(t)| \leq c : a \leq t \leq b \}$.

Case 1.

Let $0 < a < b < 1$ and $c \geq 0$. Then

$$(27) \quad P\{ |B(t)| \leq c : a \leq t \leq b \} \\ = \frac{1}{\sqrt{2\pi}} \int_0^{\mu/\sqrt{a}} \Gamma(-\mu - \sqrt{a'}h, -c, \mu - \sqrt{a'}h, \delta_1, b' - a') e^{-1/2h^2} dh,$$

where $\mu = c(1+a')$, $a' = a/(1-a)$, $b' = b/(1-b)$ and $\Gamma(\cdot)$ is defined in (14).

Anderson and Darling (1952, (5.9) on page 210) have also considered (27) and have shown that

$$(28) \quad P\{ |B(t)| \leq c : a \leq t \leq b \} \\ = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2c^2k^2} M(2kc\sqrt{a'}, 2kc\sqrt{b'}, c\sqrt{a'a}, c\sqrt{b'b}, -\sqrt{a'b'})$$

where $M(u, v, u', v', \rho)$ is the volume under the bivariate normal distribution surface with zero means, unit variances and correlation ρ which is above the rectangle with vertices $x = u \pm u'$ and $y = v \pm v'$.

Case 2.

Let $a = b$ ($\neq 0$ or 1) and $c \geq 0$. Then

$$(29) \quad P\{ |B(a)| \leq c \} = 2 \Phi [c/\sqrt{a(1-a)}] - 1.$$

GENERAL DESCRIPTION OF THE ALGORITHMS.

Overview.

The package contains algorithms for computing the probabilities associated with Wiener and Brownian bridge processes of the following forms:

$$(30) \quad \begin{aligned} &P\{ W(t) \leq \gamma : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0; \\ &P\{ |W(t)| \leq \gamma : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0; \\ &P\{ \gamma' \leq W(t) \leq \gamma : a \leq t \leq b \}, b < \infty, \gamma > 0, \gamma' < 0; \\ &P\{ W(t) \leq \gamma + \delta t : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0, \gamma + \delta b \geq 0; \\ &P\{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \}, a \geq 0, b < \infty, \gamma > 0, \gamma + \delta b \geq 0; \\ &P\{ \gamma' + \delta' t \leq W(t) \leq \gamma + \delta t : 0 \leq t \leq b \}, b < \infty, \gamma > 0, \gamma' < 0, \gamma' + \delta' b \leq \gamma + \delta b; \\ &P\{ B(t) \leq c : a \leq t \leq b \}, a \geq 0, b \leq 1, c > 0; \\ &P\{ |B(t)| \leq c : a \leq t \leq b \}, a \geq 0, b \leq 1, c > 0, \end{aligned}$$

where $W(\cdot)$ and $B(\cdot)$ are Wiener and Brownian bridge processes, respectively.

The relationships between the formulae in the previous sections and the corresponding subroutines in WIENER PACK are listed in Table 1.

From a theoretical point of view, the two routines GWPAB and GWPSAB supersede all other routines except for WIEN1 and WIENER. That is, all the other routines are special cases of these two routines. The relationships are shown in Figure 1.

However, in actual computations, each formula related to Wiener processes shown in Table 1 is programmed separately for computational efficiency.

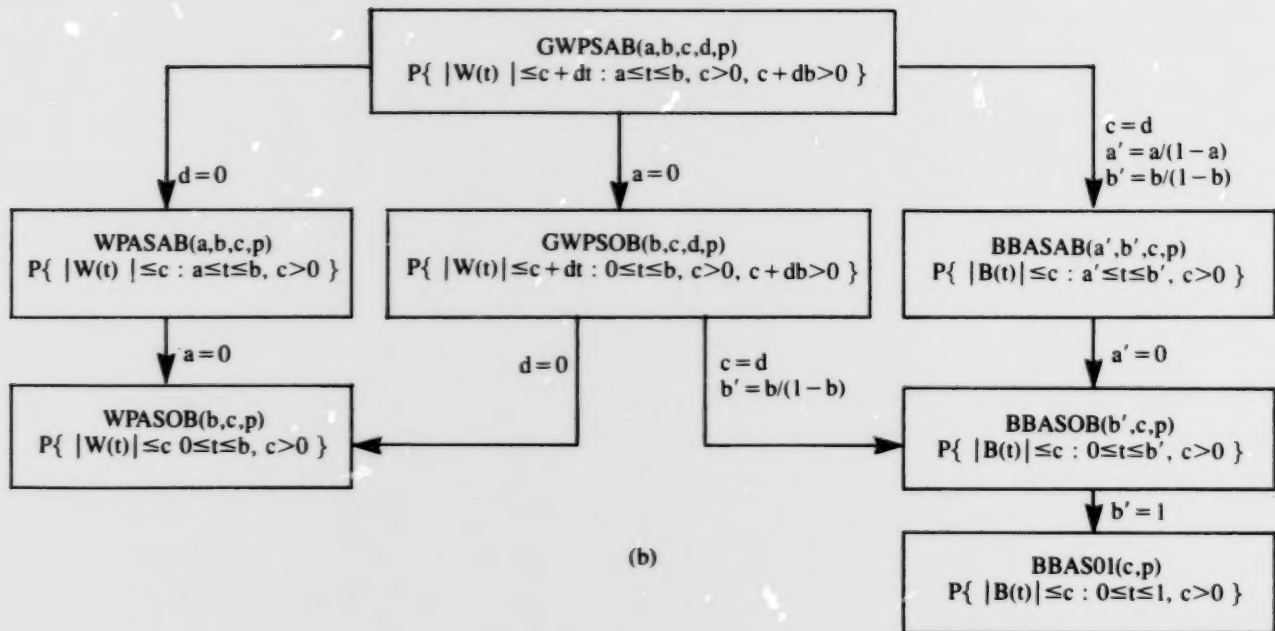
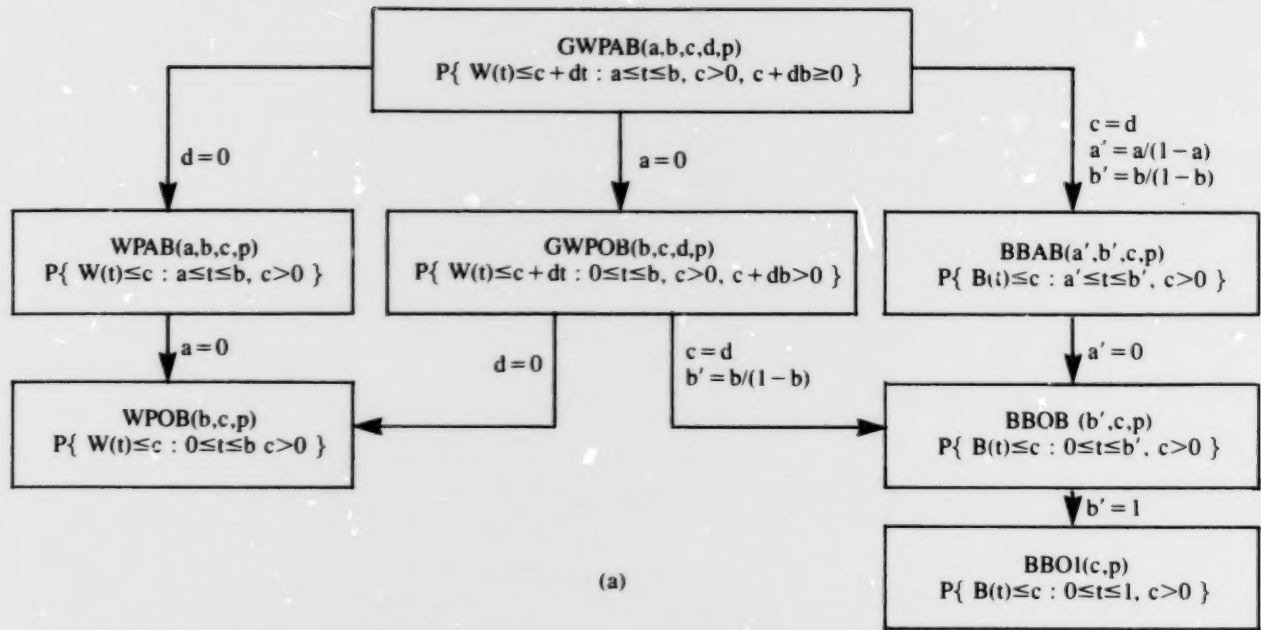
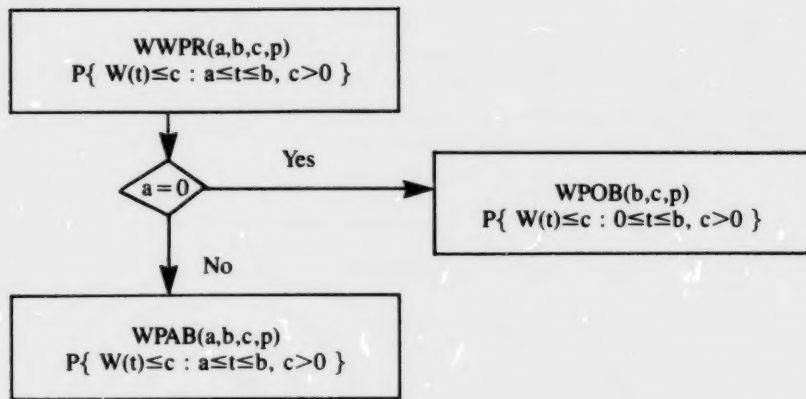
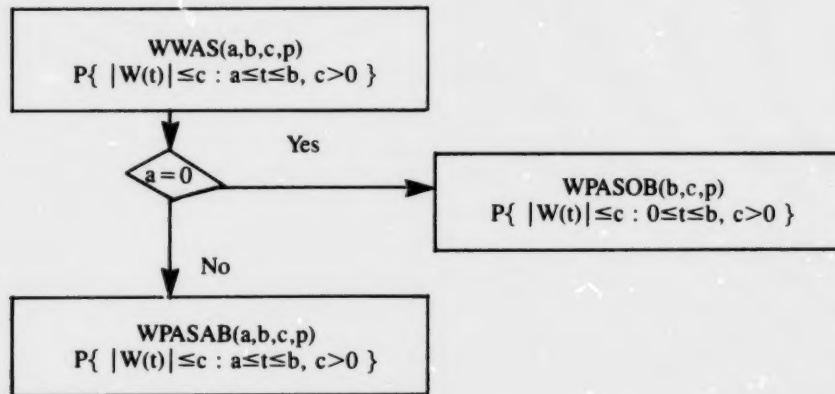


Figure 1. Interrelationships among the routines. The lower routines are the special cases of the original, upper routines and the equations beside the arrows indicate the relationships.



(a)

Figure 2. (a). General flowchart of WWPR which combines WPOB and WPAB. It calls WPOB or WPAB depending upon the input values a and b .



(b)

(b). General flowchart of WWAS which combines WPASOB and WPASAB. It calls WPASOB or WPASAB depending upon the input values a and b .

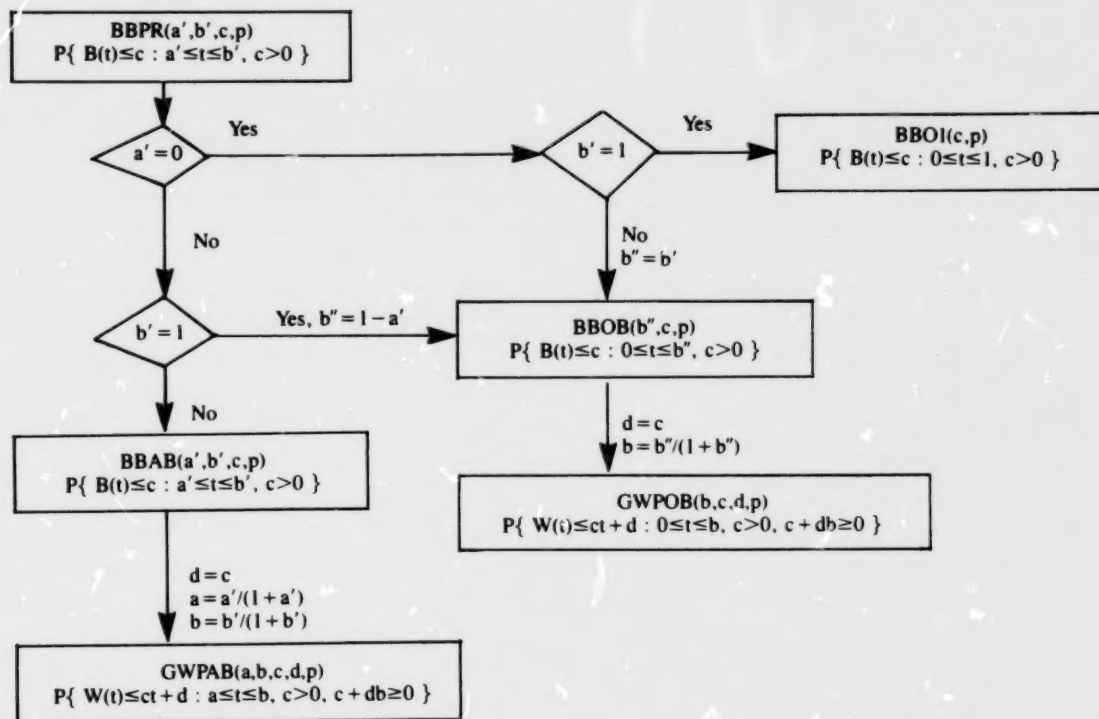


Figure 2(c). General flowchart of BBPR which combines BBO1, BBOB and BBAB. It calls BBO1, BBOB or BBAB depending upon the input value a' and b' . It also illustrates how routines GWPOB and GWPAB are utilized for computing BBOB and BBAB, respectively.

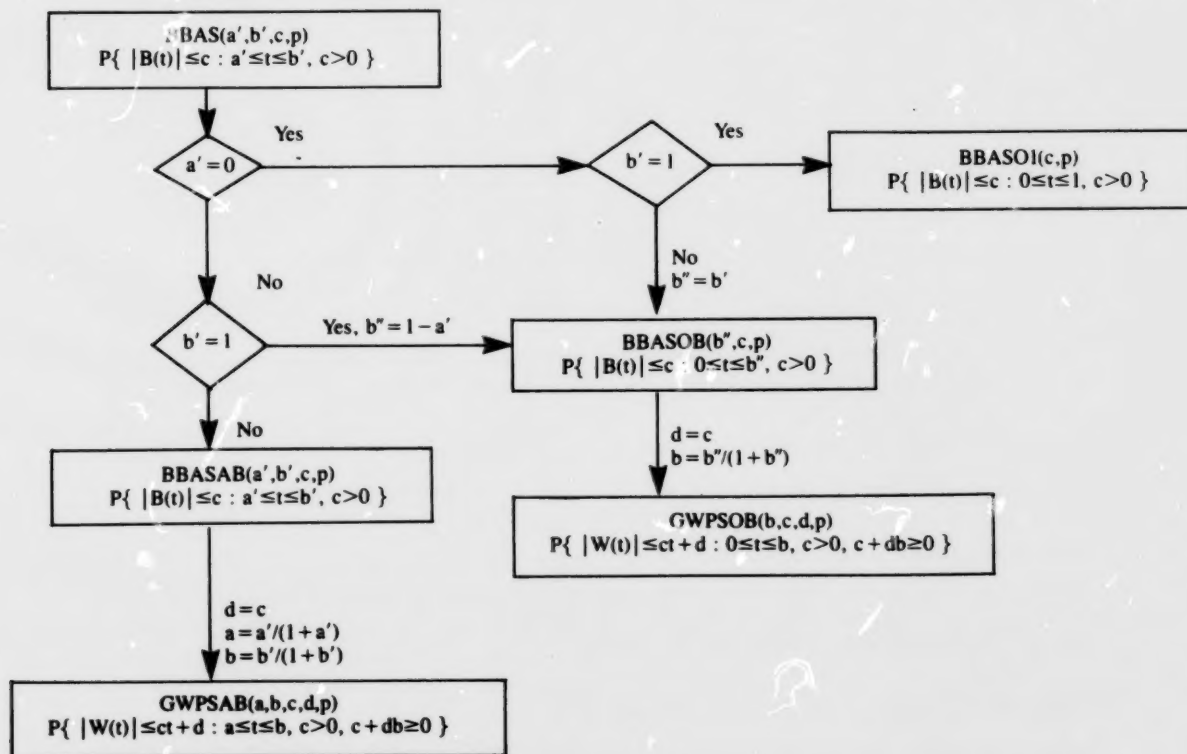


Figure 2(d). General flowchart of BBAS which combines BBASO1, BBASOB and BBASAB. It calls BBASO1, BBASOB or BBASAB depending upon the input value a' and b' . It also illustrates how routines GWPSOB and GWPSAB are utilized for computing BBASOB and BBASAB, respectively.

Four routines were added to the package for easy access to the subroutines: WWPR for combining WWOB and WWAB, WWAS for WWASOB and WWASAB, BBPR for BBO1, BBOB and BBAB, and BBAS for BBASO1, BBASOB and BBASAB, and these relationships are illustrated in Figure 2.

For the numerical integrations required for the routines WPAB, WPASAB, GWPAB and GWPSAB in this package, subroutine QNG of QUADPACK (Piessens et al., 1983) has been used. See QUADPACK (p. 130-136) for the detailed description of QNG.

In addition, two routines INVWW and INVBB have been added to compute one of parameters selected for a given probability using the four routines WWPR, WWAS, BBPR and BBAS.

Consider p_1, p_2, p_3 and p_4 as functions of three parameters a, b and c in the following four formulae:

$$\begin{aligned} p_1(a,b,c) &= P\{ W(t) \leq c : a \leq t \leq b \}, \\ p_2(a,b,c) &= P\{ |W(t)| \leq c : a \leq t \leq b \}, \\ (31) \quad p_3(a,b,c) &= P\{ B(t) \leq c : a \leq t \leq b \} \\ p_4(a,b,c) &= P\{ |B(t)| \leq c : a \leq t \leq b \}. \end{aligned}$$

Chung (1986) have shown that p_1, p_2, p_3 , and p_4 are monotonically non-decreasing and continuous functions with respect to c, a and $-b$. Subsequently the following functions can be defined:

$$(32) \quad q_k(a,b,p) := \{ c : p_k(a,b,c) = p \},$$

$$(33) \quad r_k(b,c,p) := \{ a : p_k(a,b,c) = p \},$$

$$(34) \quad s_k(a,c,p) := \{ b : p_k(a,b,c) = p \},$$

for $k = 1, 2, 3$ and 4 . Then using these monotonicity, and the routines WWPR, WWAS, BBPR and BBAS, we can iteratively compute the values of $q_k(a,b,p)$ for given values for a, b and p , the values of $r_k(b,c,p)$ for given values for b, c and p and the values of $s_k(a,c,p)$ for given values for a, c and p for $k = 1, 2, 3$ and 4 . These iterative procedures are shown in Figures 3 and 4 for INVWW and INVBB – the former for Wiener process and the latter for Brownian bridge process.

Routines for Wiener process.

WPOB (b, γ, p): $p = P\{ W(t) \leq \gamma : 0 \leq t \leq b \}$.

Given two parameters b and γ , this routine will compute and return the probability p .

WPAB (a, b, γ, p): $p = P\{ W(t) \leq \gamma : a \leq t \leq b \}$.

Given the three parameters a, b and γ , this routine will compute and return the probability p . The integrand in (5) is computed by FUNCTION WFUNC1 and QNG is called the numerical integration.

WPASOB (b, γ, p): $p = P\{ |W(t)| \leq \gamma : 0 \leq t \leq b \}$.

Given parameters b and γ , this routine will compute and return the probability p .

WIEN1 (γ', γ, b, p): $p = P\{ \gamma' \leq W(t) \leq \gamma : 0 \leq t \leq b \}$.

Given three parameters b, γ and γ' , this routine will compute and return probability p . If $\gamma = \gamma'$, then it does provide the same probability p as that obtained by WPAB (b, γ, p). This routine is used in the following routine WPASAB through FUNCTION WFUNC2.

WPASAB (a, b, γ, p): $p = P\{ |W(t)| \leq \gamma : a \leq t \leq b \}$.

Given parameters a, b and γ , this routine will compute and return probability p . FUNCTION WFUNC2 evaluates the values for the integrand function in (16); the probability argument in the integrand is evaluated by WIEN1 and QNG is called the numerical integration.

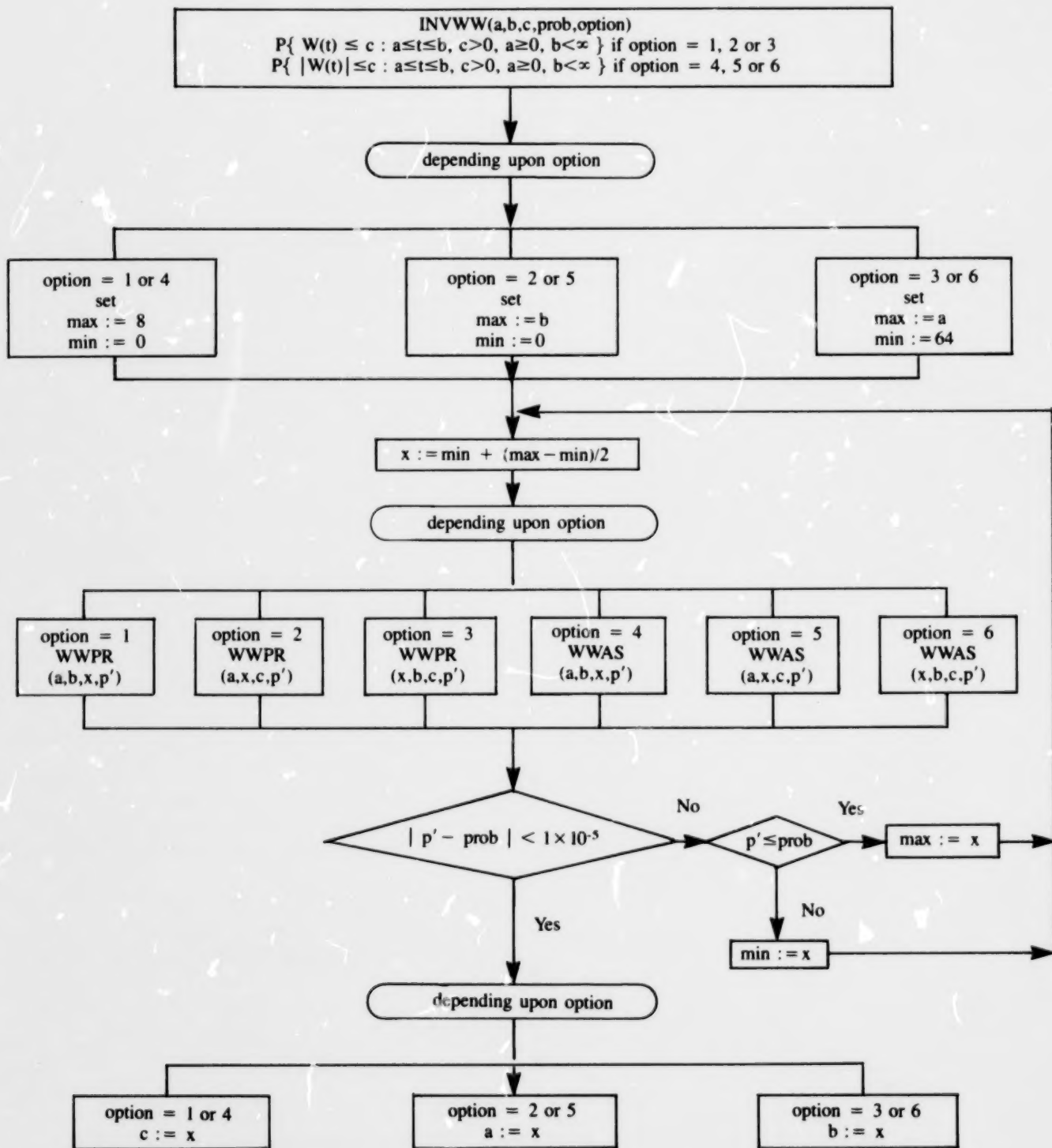


Figure 3. System flowchart for INVWW. It illustrates how the iterative procedure is used and the routines WWPR and WWAS are called for computations.

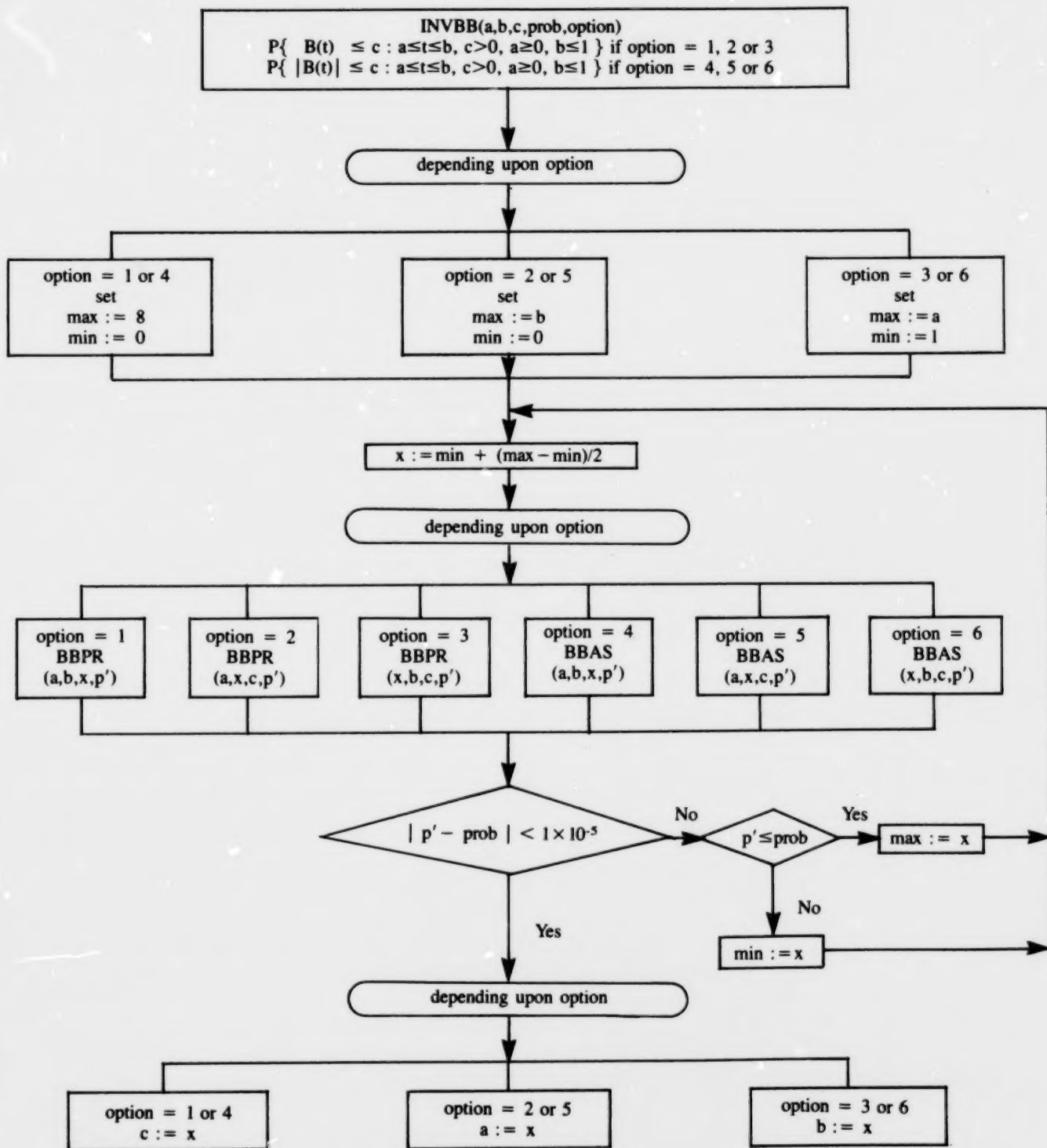


Figure 4. System flowchart for INVBB. It illustrates how the iterative procedure is used and the routines BBPR and BBAS are called for computations.

GWPOB (b, γ , δ , p): $p = P\{ W(t) \leq \gamma + \delta t : 0 \leq t \leq b \}$.

Given parameters b, γ and δ , this routine will compute and return probability p. It is used by routines GWPAB and BBOB.

GWPAB (a,b, γ , δ ,p): $p = P\{ W(t) \leq \gamma + \delta t : a \leq t \leq b \}$.

Given parameters a, b, γ , and δ , this routine will compute and return probability p. FUNCTION WFUNC3 evaluates the values for the integrand function in (4), but the probability argument in the integrand is computed by GWPOB. QNG is called the numerical integration.

GWPSOB (b, γ , δ ,p): $P\{ |W(t)| \leq \gamma + \delta t : 0 \leq t \leq b \}$.

Given the three parameters b, γ and δ , this routine will compute and return probability p. It is used by BBASOB. It is also a special case of WIENER when $\gamma = -\gamma'$ and $\delta = -\delta'$ in WIENER.

WIENER (γ' , δ' , γ , δ ,b,p): $P\{ \gamma' + \delta' t \leq W(t) \leq \gamma + \delta t : 0 \leq t \leq b \}$

Given five parameters (b, γ , δ , γ' , and δ'), this routine will compute and return probability p. It has been utilized by GWPSAB.

GWPSAB (a,b, γ , δ ,p): $P\{ |W(t)| \leq \gamma + \delta t : a \leq t \leq b \}$.

With the four parameters a, b, γ and δ , this routine will compute and return probability p. FUNCTION WFUNC4 evaluates the values for the integrand function in (15); the probability argument in the integrand is evaluated by WIENER and QNG is called the numerical integration. It is used by BBASAB.

Routines for Brownian bridge process.

BBO1 (c,p): $p = P\{ B(t) \leq c : 0 \leq t \leq 1 \}$.

For a given parameter c, this routine will compute and return probability p.

BBOB(b,c,p): $p = P\{ B(t) \leq c : 0 \leq t \leq b \}$.

For two parameters b and c, this routine will compute and return probability p. In this routine, we first set $b' := b/(1-b)$ and then evaluate GWPOB(b' ,c,c,p).

BBAB (a,b,c,p): $p = P\{ B(t) \leq c : a \leq t \leq b \}$.

Given the three parameters a, b and c, this routine will compute and return probability p. Similar to BBOB, we set $a' := a/(1-a)$, $b' := b/(1-b)$ and then evaluate GWPAB(a' , b' ,c,c,p).

BBASO1(c,p): $p = P\{ |B(t)| \leq c : 0 \leq t \leq 1 \}$.

For a given parameter c, this routine will compute and return probability p. It is the probability for the two-sided Kolmogorov-Smirnov statistics.

BBASOB(b,c,p): $p = P\{ |B(t)| \leq c : 0 \leq t \leq b \}$.

For the two parameters b and c, this routine will compute and return probability p. Set $b' := b/(1-b)$ and then call GWPSOB(b' ,c,c,p).

BBASAB(a,b,c,p): $P\{ |B(t)| \leq c : a \leq t \leq b \}$.

Given the three parameters a, b and c, this routine will compute and return the probability p. Set $a' := a/(1-a)$, $b' := b/(1-b)$ and then call GWPSAB(a' , b' ,c,c,p).

Access routines: WWPR, WWAS, BBPR, BBAS.

These four routines have been added to permit easier access to the routines discussed in the preceding sections regardless of the values of the parameters a and b . In these routines, checks are made for simple errors such as $a > b$ and error code numbers are specified and returned to the user where errors occur.

Figure 2 presents the general flowcharts of WWPR, WWAS, BBPR and BBAS and illustrates how the routines in the previous sections are called in these routines.

WWPR(a,b, γ ,p,error): $P\{W(t) \leq \gamma : a \leq t \leq b\}$.

Depending upon the input value of a , either WPOB or WPAB is called in this routine. If $a < 0$ or $a > b$, error is set equal to 101.

If $a = b = 0$, then $p := 1$. If $a = b = 0$, the $p := \Phi(\gamma/\sqrt{b})$ where $\Phi(\cdot)$ is the normal distribution function defined in section "Formulae for $p = P\{W(t) \leq \gamma + \delta t : 0 \leq t \leq b\}$."

WWAS(a,b, γ ,p,error): $P\{|W(t)| \leq \gamma : a \leq t \leq b\}$.

Depending upon the input value of a , either WPASOB or WPASAB is called in this routine. If $a < 0$ or $a > b$, error is set equal to 101.

If $a = b = 0$, $p := 0$. If $a = b = 0$, then $p := 2 \Phi(\gamma/\sqrt{b}) - 1$.

BBPR(a,b,c,p,error): $P\{B(t) \leq c : a \leq t \leq b\}$.

Depending upon the values of a and b , it calls BBO1, BBOB or BBAB. Thus this routine can be used instead of BBO1, BBOB and BBAB by specifying the proper values of a and b . If $a < 0$, $b > 1$ or $a > b$, error is set equal to 102.

If $a = b = 0$ or $a = b = 1$, then $p := 1$. If $a = b (= 0 \text{ or } 1)$, $p := \Phi(c/\sqrt{b(1-b)})$.

BBAS(a,b,c,p,error): $P\{|B(t)| \leq c : a \leq t \leq b\}$.

Depending upon the values of a and b , this routine calls BBASO1, BBASOB or BBASAB, and thus it can be used instead of BBASO1, BBASOB and BBASAB by specifying the proper values for a and b .

If $a = b (= 0 \text{ or } 1)$, then $p := 2 \Phi(c/\sqrt{b(1-b)})$.

Inverse procedures: INVWW, INVBB.

In INVWW, depending upon the value of the option specified, one of the parameters a , b or γ in either $p = P\{W(t) \leq \gamma : a \leq t \leq b\}$ (if option = 1, 2, or 3) or

$p = P\{|W(t)| \leq \gamma : a \leq t \leq b\}$ (if option = 4, 5, or 6)

is iteratively computed for a given p and the other two parameters (of a , b or γ) using the routines shown in Table 1 through WWPR and WWAS.

In INVBB, depending upon the value of the option specified, one of the parameters a , b or c in either $p = P\{B(t) \leq c : a \leq t \leq b\}$ (if option = 1, 2, or 3) or

$p = P\{|B(t)| \leq c : a \leq t \leq b\}$ (if option = 4, 5, or 6)

is iteratively computed for a given p and the other two parameters (of a , b or c) using the routines shown in Table 1 through BBPR and BBAS.

INVWW(a,b,c,p,option).

As shown in Table 2, one of the parameters is computed in this routine depending upon the value specified for option for a given probability.

The maximum possible values for c and b are set equal to 8 and 64, respectively, instead of infinity. The iteration is stopped if the absolute difference between the probability specified and the probability computed using the estimated parameter is less than 0.00001.

Table 1. Names of routines in WIENER PACK with corresponding formulae and equation numbers used in text.

Routine name	Formula	Equation no.
WPOB	$P\{W(t) \leq c : 0 \leq t \leq b, c > 0, b < \infty\}$	3
WPAB	$P\{W(t) \leq c : a \leq t \leq b, c > 0, a \geq 0, b < \infty\}$	5
WPASOB	$P\{ W(t) \leq c : 0 \leq t \leq b, c > 0, b < \infty\}$	12
WPASAB	$P\{ W(t) \leq c : a \leq t \leq b, c > 0, a \geq 0, b < \infty\}$	16
GWPOB	$P\{W(t) \leq c + dt : 0 \leq t \leq b, c > 0, c + db \geq 0, b < \infty\}$	2
GWPAAB	$P\{W(t) \leq c + dt : a \leq t \leq b, c > 0, c + db \geq 0, a \geq 0, b < \infty\}$	4
GWPSOB	$P\{ W(t) \leq c + dt : 0 \leq t \leq b, c > 0, c + db \geq 0, b < \infty\}$	9
GWPSAB	$P\{ W(t) \leq c + dt : a \leq t \leq b, c > 0, c + db \geq 0, a \geq 0, b < \infty\}$	15
WIEN1	$P\{c' \leq W(t) \leq c : 0 \leq t \leq b, c > 0, c' < 0, b < \infty\}$	10
WIENER	$P\{c' + d't \leq W(t) \leq c + dt : 0 \leq t \leq b, c > 0, c + db \geq 0, c' < 0, c' + d'b \leq 0, b < \infty\}$	7
BBO1	$P\{B(t) \leq c : 0 \leq t \leq 1, c > 0\}$	21
BBOB	$P\{B(t) \leq c : 0 \leq t \leq b, c > 0, b \leq 1\}$	20
BBAB	$P\{B(t) \leq c : a \leq t \leq b, c > 0, a \geq 0, b \leq 1\}$	22
BBASO1	$P\{ B(t) \leq c : 0 \leq t \leq 1, c > 0\}$	26
BBASOB	$P\{ B(t) \leq c : 0 \leq t \leq b, c > 0, b \leq 1\}$	25
BBASAB	$P\{ B(t) \leq c : a \leq t \leq b, c > 0, a \geq 0, b \leq 1\}$	27

Table 2. Options for INVWW.

Option	formula	parameters specified	parameter computed
1	$P\{W(t) \leq c : a \leq t \leq b\}$	a, b	c
2	$P\{W(t) \leq c : a \leq t \leq b\}$	b, c	a
3	$P\{W(t) \leq c : a \leq t \leq b\}$	a, c	b
4	$P\{ W(t) \leq c : a \leq t \leq b\}$	a, b	c
5	$P\{ W(t) \leq c : a \leq t \leq b\}$	b, c	a
6	$P\{ W(t) \leq c : a \leq t \leq b\}$	a, c	b

Table 3. Options for INVBB.

Option	formula	parameters specified	parameter computed
1	$P\{B(t) \leq c : a \leq t \leq b\}$	a, b	c
2	$P\{B(t) \leq c : a \leq t \leq b\}$	b, c	a
3	$P\{B(t) \leq c : a \leq t \leq b\}$	a, c	b
4	$P\{ B(t) \leq c : a \leq t \leq b\}$	a, b	c
5	$P\{ B(t) \leq c : a \leq t \leq b\}$	b, c	a
6	$P\{ B(t) \leq c : a \leq t \leq b\}$	a, c	b

Through WWPR and WWAS, the routines discussed in section on the Wiener process are iteratively utilized for this routine.

INVBB.

As shown in Table 3, one of the parameters is computed in this routine depending upon the value specified for option for a given probability.

The maximum possible value for c is set equal to 8 instead of infinity. The iteration is stopped if the absolute difference between the probability specified and the probability computed using the estimated parameter is less than 0.00001.

Through BBPR and BBAS, the routines discussed in the section on the Brownian bridge process are iteratively utilized for this routine.

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APPENDIX

Listing of the routines

In addition to the routines discussed in the text, a main program (MAIN) and four subroutines (WWIN, BBIN, INWW and INBB) are included in this listing. The main program and the four subroutines allow the user to access the routines discussed interactively.

An IBM-PC formatted diskette containing the source code of the routines is included in Geological Survey of Canada Open File 1426. A limited number of copies of this Open File can be purchased from

Geological Survey of Canada,
601 Booth Street, Ottawa, Ontario
Canada, K1A 0E8

```

PROGRAM GSC
C
C MAIN PROGRAM GSC
C
C COMPUTING PROBABILITY FOR GIVEN PARAMETERS AND PARAMETERS
C FOR GIVEN PROBABILITY OF WIENER PROCESS, BROWNIAN BRIDGE
C PROCESS.
C
C GSC PAPER 86-XX
C
C COMMON /FUNCT/DELTA(10)
C
C WRITE(*,999)' THIS PROGRAM HAS BEEN DEVELOPED AT THE '
C WRITE(*,999)' GEOLOGICAL SURVEY OF CANADA.'
C WRITE(*,999)
10 WRITE(*,999)' THE FOLLOWING OPTIONS ARE AVAILABLE:'
C WRITE(*,999)' 1 : COMPUTE PROBABILITY OF A WIENER PROCESS'
C WRITE(*,999)' 2 : COMPUTE PROBABILITY OF A BROWNIAN BRIDGE'
C WRITE(*,999)' 3 : GIVEN PROBABILITY LEVEL, COMPUTE PARAMETER'
C WRITE(*,999)' SELECTED OF A WIENER PROCESS'
C WRITE(*,999)' 4 : GIVEN PROBABILITY LEVEL, COMPUTE PARAMETER'
C WRITE(*,999)' SELECTED OF A BROWNIAN BRIDGE'
C WRITE(*,999)' 5 : STOP.'
C WRITE(*,999)
20 WRITE(*,999)' ENTER OPTION (1-5)'
C READ(*,998)ISEL
999 FORMAT(A)
998 FORMAT(I1)
C IF (ISEL.LT.0.OR.ISEL.GT.5)GO TO 20
C GOTO (110,120,130,140,150),ISEL
110 CALL WWIN
C GO TO 10
120 CALL BBIN
C GO TO 10
130 CALL INNW
C GO TO 10
140 CALL INNB
C GO TO 10
150 STOP
C END
C
C SUBROUTINE WWIN
C
C CALLING FOR COMPUTING PROBABILITY OF WIENER PROCESS
C
C CHARACTER IYES*1,ANSWER*1
C DATA IYES/'Y'/
10 WRITE(*,999)' ENTER 0 FOR P = PROB( W(T)<C : A<T<B )'
C WRITE(*,999)' OR'
C WRITE(*,999)' ENTER 1 FOR P = PROB( -C<W(T)<C : A<T<B )'
C READ(*,998)INN
C WRITE(*,999)' ENTER C ( > 0 ), C = '
C READ(*,997)C
C WRITE(*,999)' ENTER A ( >= 0 ), A = '
C READ(*,997)A
C WRITE(*,999)' ENTER B ( > A ), B = '
C READ(*,997)B
C
999 FORMAT(A)
998 FORMAT(I1)
997 FORMAT(F12.0)
996 FORMAT(A,I3)
995 FORMAT(A,F12.6)
994 FORMAT(A1)
C IF ( INN.EQ.0 ) THEN
C CALL WWPR( A,B,C,PROB,IER )
C ELSE
C CALL WWAS( A,B,C,PROB,IER )
C ENDIF
C
C IF ( IER.NE.0 ) THEN
C WRITE(*,996)' ERROR OCCURS, ERROR NUMBER = ',IER
C ELSE
C WRITE(*,995)' GIVEN PARAMETERS, PROBABILITY = ',PROB
C ENDIF
C WRITE(*,999)' WOULD YOU LIKE TO TRY ANOTHER PARAMETERS?(Y OR N)'
C READ(*,994)ANSWER
C IF( ANSWER.EQ.IYES ) GO TO 10
C RETURN
C END
C
C SUBROUTINE BBIN
C
C CALLING FOR COMPUTING PROBABILITY OF BROWIAN BRIDGE PROCESS
C
C CHARACTER IYES*1,ANSWER*1
C DATA IYES/'Y'/
10 WRITE(*,999)' ENTER 0 FOR P = PROB( B(T)<C : A<T<B )'
C WRITE(*,999)' OR'
C WRITE(*,999)' ENTER 1 FOR P = PROB( -C<B(T)<C : A<T<B )'
C READ(*,998)INN
C WRITE(*,999)' ENTER C ( > 0 ), C = '
C READ(*,997)C
C WRITE(*,999)' ENTER A ( 0<=A<B ), A = '
C READ(*,997)A
C WRITE(*,999)' ENTER B ( A<B<=1 ), B = '
C READ(*,997)B
C
999 FORMAT(A)
998 FORMAT(I1)
997 FORMAT(F12.0)
996 FORMAT(A,I3)

```

```

995 FORMAT(A,F12.6)
994 FORMAT(A1)
C IF ( INN.EQ.0 ) THEN
C CALL BBPR( A,B,C,PROB,IER )
C ELSE
C CALL BBAS( A,B,C,PROB,IER )
C ENDIF
C
C IF ( IER.NE.0 ) THEN
C WRITE(*,996)' ERROR OCCURS, ERROR NUMBER = ',IER
C ELSE
C WRITE(*,995)' GIVEN PARAMETERS, PROBABILITY = ',PROB
C ENDIF
C WRITE(*,999)' WOULD YOU LIKE TO TRY ANOTHER PARAMETERS?(Y OR N)'
C READ(*,995)ANSWER
C IF( ANSWER.EQ.IYES ) GO TO 10
C RETURN
C END
C
C SUBROUTINE INNW
C
C CALLING FOR COMPUTING PARAMETER OF WIENER PROCESS, FOR
C GIVEN PROBABILITY LEVEL
C
C CHARACTER IYES*1,ANSWER*1
C DATA IYES/'Y'/
10 WRITE(*,999)' ENTER 0 FOR P = PROB( W(T)<C : A<T<B )'
C WRITE(*,999)' OR'
C WRITE(*,999)' ENTER 1 FOR P = PROB( -C<W(T)<C : A<T<B )'
C WRITE(*,999)' OR'
C WRITE(*,999)' ENTER 2 FOR P = PROB( W(T)<C+DT : A<T<B )'
C WRITE(*,999)' OR'
C WRITE(*,999)' ENTER 3 FOR P = P( -C-DT<W(T)<C+DT : A<T<B )'
C READ(*,998)INN
C
999 FORMAT(A)
998 FORMAT(I1)
997 FORMAT(F12.0)
996 FORMAT(A,I3)
995 FORMAT(A,F12.6)
994 FORMAT(A1)
C IF ( INN.EQ.0 ) THEN
C WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 1'
C WRITE(*,999)' A 2'
C WRITE(*,999)' B 3'
C WRITE(*,999)' OPTION = '
C ENDIF
C IF ( INN.EQ.1 ) THEN
C WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 4'
C WRITE(*,999)' A 5'
C WRITE(*,999)' B 6'
C WRITE(*,999)' OPTION = '
C ENDIF
C IF ( INN.EQ.2 ) THEN
C WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 7'
C WRITE(*,999)' D 8'
C WRITE(*,999)' OPTION = '
C ENDIF
C IF ( INN.EQ.3 ) THEN
C WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 9'
C WRITE(*,999)' D 10'
C WRITE(*,999)' OPTION = '
C ENDIF
C READ(*,998)IOP
C WRITE(*,999)' ENTER P ( 0<P<1 ), P = '
C READ(*,997)P
C IF ( IOP.EQ.1.OR.IOP.EQ.4.OR.IOP.EQ.7 ) THEN
C WRITE(*,999)' ENTER A ( >= 0 ), A = '
C READ(*,997)A
C WRITE(*,999)' ENTER B ( > A ), B = '
C READ(*,997)B
C IF ( IOP.EQ.7.OR.IOP.EQ.9 ) THEN
C WRITE(*,999)' ENTER D , D = '
C READ(*,997)D
C ELSE
C WRITE(*,999)' ENTER C , C = '
C READ(*,997)C
C ENDIF
C ELSE
C IF ( IOP.EQ.2.OR.IOP.EQ.5 ) THEN
C WRITE(*,999)' ENTER C ( > 0 ), C = '
C READ(*,997)C
C WRITE(*,999)' ENTER B ( > A ), B = '
C READ(*,997)B
C ELSE
C WRITE(*,999)' ENTER C ( > 0 ), C = '
C READ(*,997)C
C WRITE(*,999)' ENTER A ( >= 0 ), A = '
C READ(*,997)A
C ENDIF
C ENDIF
C CALL INVW( A,B,C,D,F,IOP,IER )
C
C IF ( IER.NE.0 ) THEN
C WRITE(*,996)' ERROR OCCURS, ERROR NUMBER = ',IER
C ELSE
C IF ( IOP.EQ.1.OR.IOP.EQ.4.OR.IOP.EQ.7.OR.IOP.EQ.9 ) THEN
C PARA = C
C ELSE
C IF ( IOP.EQ.2.OR.IOP.EQ.5 ) THEN
C PARA = A
C ELSE

```

```

      IF ( IOP.EQ.3.OR.IOP.EQ.6 ) THEN
        PARA = B
      ELSE
        PARA = D
      ENDIF
    ENDIF
  ENDIF
  WRITE(*,995) ' GIVEN PROBABILITY, PARAMETER = ',PARA
  ENDIF
  WRITE(*,999)' WOULD YOU LIKE TO TRY ANOTHER PARAMETERS?(Y OR N)'
  READ(*,994)ANSWER
  IF( ANSWER.EQ.IYES ) GO TO 10
  RETURN
END
C=====
C
      SUBROUTINE INBB
C
C CALLING FOR COMPUTING PARAMETER OF BROWNIAN BRIDGE PROCESS,
C FOR GIVEN PROBABILITY LEVEL
C
      CHARACTER IYES*1,ANSWER*1
      DATA IYES/'Y'/
10    WRITE(*,999)' ENTER 0 FOR P = PROB( B(T)<C : A<T<B )'
      WRITE(*,999)' OR'
      WRITE(*,999)' ENTER 1 FOR P = PROB( -C<B(T)<C : A<T<B )'
      READ(*,998)INN
C
999  FORMAT(A)
998  FORMAT(I1)
997  FORMAT(F12.0)
996  FORMAT(A,I3)
994  FORMAT(A,F12.6)
994  FORMAT(A1)
      IF ( INN.EQ.0 ) THEN
        WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 1'
        WRITE(*,999)' A 2'
        WRITE(*,999)' B 3'
        WRITE(*,999)' OPTION = '
      ELSE
        WRITE(*,999)' TO CALCULATE C , ENTER OPTION = 4'
        WRITE(*,999)' A 5'
        WRITE(*,999)' B 6'
        WRITE(*,999)' OPTION = '
      ENDIF
      READ(*,998)IOP
      WRITE(*,999)' ENTER P ( 0<P<1 ) , P = '
      READ(*,997)P
      IF ( IOP.EQ.1.OR.IOP.EQ.4 ) THEN
        WRITE(*,999)' ENTER A ( >=0 ) , A = '
        READ(*,997)A
        WRITE(*,999)' ENTER B ( > A ) , B = '
        READ(*,997)B
      ELSE
        IF ( IOP.EQ.2.OR.IOP.EQ.5 ) THEN
          WRITE(*,999)' ENTER C ( > 0 ) , C = '
          READ(*,997)C
          WRITE(*,999)' ENTER B ( > A ) , B = '
          READ(*,997)B
        ELSE
          WRITE(*,999)' ENTER C ( > 0 ) , C = '
          READ(*,997)C
          WRITE(*,999)' ENTER A ( >=0 ) , A = '
          READ(*,997)A
        ENDIF
      ENDIF
      CALL INVBB( A,B,C,P,IOP,IER )
C
C
      IF ( IER.NE.0 ) THEN
        WRITE(*,996)' ERROR OCCURS, ERROR NUMBER = ',IER
      ELSE
        IF ( IOP.EQ.1.OR.IOP.EQ.4 ) THEN
          PARA = C
        ELSE
          IF ( IOP.EQ.2.OR.IOP.EQ.5 ) THEN
            PARA = A
          ELSE
            PARA = B
          ENDIF
        ENDIF
        WRITE(*,995) ' GIVEN PROBABILITY, PARAMETER = ',PARA
      ENDIF
      WRITE(*,999)' WOULD YOU LIKE TO TRY ANOTHER PARAMETERS?(Y OR N)'
      READ(*,994)ANSWER
      IF( ANSWER.EQ.IYES ) GO TO 10
      RETURN
END
C=====
C
      SUBROUTINE FLOW ( UFLOW,OFLOW )
C=====
C
C THIS ROUTINE PROVIDES THE MACHINE CONSTANTS OF THE
C SMALLEST AND LARGEST NUMBERS.
C
C PARAMETERS:
C OUT UFLOW : THE SMALLEST POSITIVE NUMBER
C OFLOW : THE LARGEST POSITIVE NUMBER
C
C=====
      UFLOW = 1.0E-37
      OFLOW = 1.0E+38
      RETURN

```

```

      END
C=====
C
      REAL FUNCTION PHI(X)
C=====
C
C 1. PHI - FORTRAN 77 REAL FUNCTION
C 2. IT COMPUTES THE PROBABILITY OF STANDARD NORMAL DISTRIBUTION
C FUNCTION:
C PHI(X) = PROB( N < X ) WHERE N ~ NORMAL( 0, 1 )
C 3. PARAMETERS:
C IN X : SEE THE ABOVE FORMULA
C OUT PHI : PROBABILITY
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C NONE
C 5. REFERENCES:
C ABRAMOWITZ AND STEGUN ( 1964, (26.2.17) OF P.932 )
C=====
      DIMENSION B(5)
      DATA B/ 0.319381530, -0.356563782, 1.781477937,
      C -1.821255978, 1.330274429/
      DATA PP/0.231641900/, PH/3.14159265359/
C
      IN=0
      IF(X.LT.0.0)THEN
        IN=1
        X=-X
      ENDIF
      M=5
      CALL FLOW ( UFLOW, OFLOW )
      EUFLOW = -ALOG( UFLOW )
      XOM = X*X/2.0
      IF(XOM.LT.EUFLOW)THEN
        T=1.0/(1.0+PP*X)
        ZX=(EXP(-XOM))/SQRT(2.0*PH)
        TT=1.
        BT=0.
        DO 10 I=1,M
          TT=TT*T
          BT=BT+B(I)*TT
10      CONTINUE
      ELSE
        ZX=0.
        BT=0.
      ENDIF
      PHI=1.0-ZX*BT
      IF(IN.EQ.1)THEN
        PHI=1.0-PHI
        X=-X
      ENDIF
      RETURN
      END
C=====
C
      SUBROUTINE WWFR(A,B,C,PROB,IER)
C=====
C
C 1. WWFR - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C PROB = P { W ( T ) < C : A < T < B }
C
C 3. PARAMETERS:
C IN A : SEE THE ABOVE FORMULA
C B : "
C C : "
C OUT PROB : "
C IER : ERROR AND IER > 0, IF AN ERROR OCCURS.
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - WWOB = PHI
C - WwAB = QNG - QMACO
C - WFUNC1 = PHI
C=====
C
C * * * * * CHECK ON THE VALIDITY OF PARAMETERS
C
      IF ( A.LT.0.0 .OR. A.GT.B ) THEN
        IER=101
        PROB=0.0
      ELSE
        IER=0
        IF ( A.EQ.B ) THEN
          C * * * * * SET PROB = PHI( C/SQRT(B) ), IF A = B.
          IF ( B.EQ.0. ) THEN
            PROB = 1.0
          ELSE
            BB = SQRT( B )
            PROB = PHI ( C / BB )
          ENDIF
        ELSE
          IF ( A.EQ.0.0 ) THEN
            C * * * * * CALL WPOB TO COMPUTE P { W(T) < C : 0 < T < B }
            CALL WPOB( B,C,PROB )
          ELSE

```

```

C ***** CALL WPAB TO COMPUTE P { W(T) < C : A < T < B }
C
      CALL WPAB( A,B,C,PROB )
      ENDIF
      ENDIF
      IF ( PROB.GE.100. ) THEN
        IER = PROB
        PROB = 0.
      ENDIF
      ENDIF
      RETURN
      END

```

```

C
C SUBROUTINE WWAS(A,B,C,PROB,IER)
C*****
C 1. WWAS - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C    PROB = P { -C < W(T) < C : A < T < B }
C
C 3. PARAMETERS:
C    IN  A : SEE THE ABOVE FORMULA
C        B :
C        C :
C    OUT PROB :
C        IER : ERROR AND IER > 0, IF AN ERROR OCCURS.
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - WWASOB - FLOW
C    - WWASAB - QNG - QMACO
C    - WFUNC2 - WIEN1 - PHI
C*****

```

```

C ***** CHECK ON THE VALIDITY OF PARAMETERS
C
      IF ( A.LT.0.0 .OR. A.GT.B ) THEN
        IER=101
        PROB=0.0
      ELSE
        IER=0
        IF ( A.EQ.B ) THEN
          C ***** SET PROB = 2*PHI( C/SQRT(B) ) - 1, IF A = B.
          C
            IF ( B.EQ.0. ) THEN
              PROB = 1.0
              BB = SQRT( B )
              PROB = 2.0 * PHI( C/BB ) - 1.0
            ENDIF
            ELSE
              IF ( A.EQ.0.0 ) THEN
                C ***** CALL WPASOB TO COMPUTE P { -C < W(T) < C : 0 < T < B }
                C
                  CALL WPASOB( B,C,PROB )
                  ELSE
                C ***** CALL WPASAB TO COMPUTE P { -C < W(T) < C : A < T < B }
                C
                  CALL WPASAB( A,B,C,PROB )
                  ENDIF
                  ENDIF
                  IF ( PROB.GE.100. ) THEN
                    IER = PROB
                    PROB = 0.
                  ENDIF
                  ENDIF
                  RETURN
                  END

```

```

C
C SUBROUTINE BBPR(A,B,C,PROB,IER)
C*****
C 1. BBPR - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:
C
C    PROB = P { B(T) < C : A < T < B }
C
C 3. PARAMETERS:
C    IN  A : SEE THE ABOVE FORMULA
C        B :
C        C :
C    OUT PROB :
C        IER : ERROR AND IER > 0, IF AN ERROR OCCURS.
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - BB01
C    - BB0B - GWP0B - PHI
C    - BBAB - GWPAB - QNG - QMACO
C    - WFUNC3 - GWP0B - PHI
C*****
C ***** CHECK ON THE VALIDITY OF PARAMETERS
C
      IF ( A.LT.0.0 .OR. A.GT.B ) THEN
        IER=102
        PROB=0.0
      ELSE
        IER=0
        IF ( A.EQ.B ) THEN
          C ***** SET PROB = 2*PHI( C/SQRT(B*(1-B)) ) - 1, IF A = B.
          C
            IF ( B.EQ.0.0 .OR. B.EQ.1.0 ) THEN
              PROB = 1.0
            ELSE
              BB = SQRT( B*(1.0-B) )
              PROB = 2.0*PHI( C/BB ) - 1.0
            ENDIF
            ELSE
              IF ( A.EQ.0.0 ) THEN
                IF ( B.EQ.1.0 ) THEN
                  C ***** CALL BB01 TO COMPUTE P { B(T) < C : 0 < T < 1 }
                  C
                    CALL BB01( C,PROB )
                    ELSE
                  C ***** CALL BB0B TO COMPUTE P { B(T) < C : 0 < T < B }
                  C
                    CALL BB0B( B,C,PROB )
                    ENDIF
                    IF ( B.EQ.1.0 ) THEN
                      BB = 1.0 - A
                    C ***** CALL BB0B TO COMPUTE P { B(T) < C : A < T < 1 }
                    C
                      CALL BB0B( BB,C,PROB )
                      ELSE
                    C ***** CALL BBAB TO COMPUTE P { B(T) < C : A < T < B }
                    C
                      CALL BBAB( A,B,C,PROB )
                      ENDIF
                      ENDIF
                      IF ( PROB.GE.100. ) THEN
                        IER = PROB
                        PROB = 0.
                      ENDIF
                      ENDIF
                      RETURN
                      END

```

```

      IF ( A.LT.0.0 .OR. B.GT.1.0 .OR. A.GT.B ) THEN
        IER=102
        PROB=0.0
      ELSE
        IER=0
        IF ( A.EQ.B ) THEN
          C ***** SET PROB = PHI( C/SQRT(B*(1-B)) ), IF A = B.
          C
            IF ( B.EQ.0.0 .OR. B.EQ.1.0 ) THEN
              PROB = 1.0
            ELSE
              BB = SQRT( B*(1.0-B) )
              PROB = PHI( C/BB )
            ENDIF
            ELSE
              IF ( A.EQ.0.0 ) THEN
                IF ( B.EQ.1.0 ) THEN
                  C ***** CALL BB01 TO COMPUTE P { B(T) < C : 0 < T < 1 }
                  C
                    CALL BB01( C,PROB )
                    ELSE
                  C ***** CALL BB0B TO COMPUTE P { B(T) < C : 0 < T < B }
                  C
                    CALL BB0B( B,C,PROB )
                    ENDIF
                    IF ( B.EQ.1.0 ) THEN
                      BB = 1.0 - A
                    C ***** CALL BB0B TO COMPUTE P { B(T) < C : A < T < 1 }
                    C
                      CALL BB0B( BB,C,PROB )
                      ELSE
                    C ***** CALL BBAB TO COMPUTE P { B(T) < C : A < T < B }
                    C
                      CALL BBAB( A,B,C,PROB )
                      ENDIF
                      ENDIF
                      IF ( PROB.GE.100. ) THEN
                        IER = PROB
                        PROB = 0.
                      ENDIF
                      ENDIF
                      RETURN
                      END

```

```

C
C SUBROUTINE BRAS(A,B,C,PROB,IER)
C*****
C 1. BRAS - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:
C
C    PROB = P { -C < B(T) < C : A < T < B }
C
C 3. PARAMETERS:
C    IN  A : SEE THE ABOVE FORMULA
C        B :
C        C :
C    OUT PROB :
C        IER : ERROR AND IER > 0, IF AN ERROR OCCURS.
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - BRAS01 - FLOW
C    - BRAS0B - GWP0B - FLOW
C    - PHI
C    - BRASAB - GWPAB - QNG - QMACO
C    - WFUNC4 - WIEN1 - FLOW
C    - PHI
C*****
C ***** CHECK ON THE VALIDITY OF PARAMETERS
C
      IF ( A.LT.0.0 .OR. B.GT.1.0 .OR. A.GT.B ) THEN
        IER=102
        PROB=0.0
      ELSE
        IER=0
        IF ( A.EQ.B ) THEN
          C ***** SET PROB = 2*PHI( C/SQRT(B*(1-B)) ) - 1, IF A = B.
          C
            IF ( B.EQ.0.0 .OR. B.EQ.1.0 ) THEN
              PROB = 1.0
            ELSE
              BB = SQRT( B*(1.0-B) )
              PROB = 2.0*PHI( C/BB ) - 1.0
            ENDIF
            ELSE
              IF ( A.EQ.0.0 ) THEN
                IF ( B.EQ.1.0 ) THEN
                  C ***** CALL BRAS01 TO COMPUTE P { -C < B(T) < C : 0 < T < 1 }
                  C
                    CALL BRAS01( C,PROB )
                    ELSE

```

```

C ***** CALL BRASOB TO COMPUTE P { -C < B(T) < C : 0 < T < B }
C
      CALL BRASOB( B,C,PROB )
      ENDIF
      ELSE
      IF ( B.EQ.1.0 ) THEN
      BB = 1.0 - A
C ***** CALL BRASOB TO COMPUTE P { -C < B(T) < C : A < T < 1 }
C
      CALL BRASOB( BB,C,PROB )
      ELSE
C ***** CALL BRASAB TO COMPUTE P { -C < B(T) < C : A < T < B }
C
      CALL BRASAB( A,B,C,PROB )
      ENDIF
      ENDIF
      IF ( PROB.GE.100. ) THEN
      IER = PROB
      PROB = 0.
      LGDIF
      ENDIF
      RETURN
      END
C=====
C SUBROUTINE INVWM( A,B,C,D,P,IOPT,IER )
C=====
C 1. INVWM - FORTRAN 77 SUBROUTINE
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE PARAMETER
C SELECTED, DEPENDING UPON THE INPUT PARAMETER IOPT, OF THE WIENER
C PROCESS. WHEN IOPT = 1, 2 OR 3, FOR GIVEN PROBABILITY P, FOR,
C
C P = PROB { W ( T ) < C : A < T < B }
C
C IT COMPUTES C , IF IOPT = 1
C " A , " = 2
C " B , " = 3
C
C WHEN IOPT = 4, 5 OR 6, FOR,
C
C P = PROB { -C < W ( T ) < C : A < T < B }
C
C IT COMPUTES C , IF IOPT = 4
C " A , " = 5
C " B , " = 6
C
C WHEN IOPT = 7 OR 8, FOR,
C
C P = PROB { W ( T ) < C+DT : A < T < B }
C
C IT COMPUTES C , IF IOPT = 7
C " D , " = 8
C
C WHEN IOPT = 9 OR 10, FOR,
C
C P = PROB { -C-DT < W ( T ) < C+DT : A < T < B }
C
C IT COMPUTES C , IF IOPT = 9
C " D , " = 10
C
C 3. PARAMETERS:
C IN IOPT : SEE THE ABOVE
C P : GIVEN PROBABILITY LEVEL
C C : IF IOPT = 2, 3, 5, 6, 8, 10
C D : " = 7, 9
C A : " = 1, 3, 4, 6, 7, 8, 9, 10
C B : " = 1, 2, 4, 5, 7, 8, 9, 10
C OUT C : IF IOPT = 1, 4, 7, 9
C D : " = 8, 10
C A : " = 2, 5
C B : " = 3, 6
C IER : ERROR CODE AND IER > 0, IF AN ERROR OCCURS.
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - WWPR - WWOB - PHI
C - WWAB - QMG - QMACO
C - WFUNC1 - PHI
C - WWAS - WWASOB - FLOW
C - WWASAB - QMG - QMACO
C - WFUNC2 - WIEN1 - PHI
C=====
C DATA XMAX/8.0/, TOL/0.000001/
C ***** CHECK ON THE VALIDITY OF PARAMETERS
C
C IF ( IOPT.LT.1 .OR. IOPT.GT.10 ) THEN
C IER=111
C RETURN
C ENDIF
C GOTO ( 10,20,30,40,50,60,70,75,80,85 ),IOPT
C ***** SET UP APPROPRIATE INITIAL VALUES
C
C 10 VMAX=XMAX
C VMIN=0.
C PROB=0.0
C CALL WWPR( A,B,VMAX,PROB,IER )

```

```

20 GO TO 90
VMAX = B
VMIN=0.0
CALL WWPR( VMIN,B,C,PROB,IER )
CALL WWPR( B,B,C,PROB,IER )
30 GO TO 90
VMAX=A
VMIN=64.
CALL WWPR( A,VMIN,C,PROB,IER )
CALL WWPR( A,A,C,PROB,IER )
GO TO 90
40 VMAX=XMAX
VMIN=0.
PROB=0.
CALL WWAS( A,B,VMAX,PROB,IER )
GO TO 90
50 VMAX=B
VMIN=0.
CALL WWAS( VMIN,B,C,PROB,IER )
CALL WWAS( B,B,C,PROB,IER )
GO TO 90
60 VMAX=A
VMIN=128.
CALL WWAS( A,VMIN,C,PROB,IER )
CALL WWAS( A,A,C,PROB,IER )
70 VMAX=XMAX
VMIN=0.
IF ( A.EQ.0.0 ) THEN
CALL GWPOB( B,VMIN,D,PROB )
CALL GWPOB( B,VMAX,D,PROB )
ELSE
CALL GWPAB( A,B,VMIN,D,PROB )
CALL GWPAB( A,B,VMAX,D,PROB )
ENDIF
GO TO 90
75 VMAX=XMAX
VMIN=0.
IF ( A.EQ.0.0 ) THEN
CALL GWPOB( B,C,VMIN,PROB )
CALL GWPOB( B,C,VMAX,PROB )
ELSE
CALL GWPAB( A,B,C,VMIN,PROB )
CALL GWPAB( A,B,C,VMAX,PROB )
ENDIF
GO TO 90
80 VMAX=XMAX
VMIN=0.
IF ( A.EQ.0.0 ) THEN
CALL GWPSOB( B,VMIN,D,PROB )
CALL GWPSOB( B,VMAX,D,PROB )
ELSE
CALL GWPSAB( A,B,VMIN,D,PROB )
CALL GWPSAB( A,B,VMAX,D,PROB )
ENDIF
GO TO 90
85 VMAX=XMAX
VMIN=0.
IF ( A.EQ.0.0 ) THEN
CALL GWPSOB( B,C,VMIN,PROB )
CALL GWPSOB( B,C,VMAX,PROB )
ELSE
CALL GWPSAB( A,B,C,VMIN,PROB )
CALL GWPSAB( A,B,C,VMAX,PROB )
ENDIF
90 IF ( IER.NE.0 ) RETURN
C
IF ( ABS(PROB-P).LT.TOL ) THEN
GOTO ( 310,320,330,310,320,330,310,340,310,340 ), IOPT
310 C = VMIN
GO TO 390
320 A = VMIN
GO TO 390
330 B = VMIN
GO TO 390
340 D = VMIN
GO TO 390
ELSE
IF ( ABS(PROB-P).LT.TOL ) THEN
GOTO ( 360,370,380,360,370,380,360,385,360,385 ), IOPT
360 C = VMAX
GO TO 390
370 A = VMAX
GO TO 390
380 B = VMAX
GO TO 390
385 D = VMAX
GO TO 390
ENDIF
ENDIF
C
IF ( PROB.GT.P.OR.PROB.LT.P ) THEN
IER=131
RETURN
ENDIF
C ***** ITERATIVE PROCEDURE STARTS FROM HERE
C
C ITER = 1
100 VNEW = VMIN + (VMAX-VMIN)/2.0
WRITE(*,992)ITER
992 FORMAT(' + THE COMPUTATIONS ARE IN PROGRESS, ITERATION NO. =',15)
ITER = ITER + 1
C
GOTO ( 110,120,130,140,150,160,170,175,180,185 ), IOPT

```

```

C
110 CALL WWPR( A,B,VNEW,PROBN,IER )
GO TO 190
120 CALL WWPR( VNEW,B,C,PROBN,IER )
GO TO 190
130 CALL WWPR( A,VNEW,C,PROBN,IER )
GO TO 190
140 CALL WWAS( A,B,VNEW,PROBN,IER )
GO TO 190
150 CALL WWAS( VNEW,B,C,PROBN,IER )
GO TO 190
160 CALL WWAS( A,VNEW,C,PROBN,IER )
GO TO 190
170 IF ( A.EQ.0.0 ) THEN
CALL GWPOB( B,VNEW,D,PROBN )
ELSE
CALL GWPAR( A,B,VNEW,D,PROBN )
ENDIF
GO TO 190
175 IF ( A.EQ.0.0 ) THEN
CALL GWPOB( B,C,VNEW,PROBN )
ELSE
CALL GWPAR( A,B,C,VNEW,PROBN )
ENDIF
GO TO 190
180 IF ( A.EQ.0.0 ) THEN
CALL GWPSOB( B,VNEW,D,PROBN )
ELSE
CALL GWPSAB( A,B,VNEW,D,PROBN )
ENDIF
GO TO 190
185 IF ( A.EQ.0.0 ) THEN
CALL GWPSOB( B,C,VNEW,PROBN )
ELSE
CALL GWPSAB( A,B,C,VNEW,PROBN )
ENDIF
190 IF ( IER.NE.0 ) RETURN
C
C ***** IF THE DIFFERENCE IS LESS THAN THE GIVEN TOLEANCE
C THEN STOP THE ITERATIONS
C
IF ( ABS(PROBN-P).GT.TOL ) THEN
IF ( P.GT.PROBN ) THEN
VMIN = VNEW
ELSE
VMAX = VNEW
ENDIF
GO TO 100
ELSE
GOTO ( 210,220,230,210,220,230,210,240,210,240 ), IOPT
210 C = VNEW
GO TO 290
220 A = VNEW
GO TO 290
230 B = VNEW
GO TO 290
240 D = VNEW
290 CONTINUE
ENDIF
390 RETURN
END
C=====
C
SUBROUTINE INVB( A,B,C,P,IOPT,IER )
C*****
C
1. INVB - FORTRAN 77 SUBROUTINE
C
2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE PARAMETER
SELECTED, DEPENDING UPON THE INPUT PARAMETER IOPT, OF THE
BROWNIAN BRIDGE PROCESS.
WHEN IOPT = 1, 2 OR 3, FOR GIVEN PROBABILITY P,
C
P = PROB { B ( T ) < C : A < T < B }
C
IT COMPUTES C , IF IOPT = 1
" A , " = 2
" B , " = 3
C
WHEN IOPT = 4, 5 OR 6, FOR,
C
P = PROB { -C < B ( T ) < C : A < T < B }
C
IT COMPUTES C , IF IOPT = 4
" A , " = 5
" B , " = 6
C
3. PARAMETERS:
IN IOPT : SEE THE ABOVE
P : GIVEN PROBABILITY LEVEL
C : IF IOPT = 2, 3, 5 OR 6
A : = 1, 3, 4 OR 6
B : = 1, 2, 3, 4 OR 5
OUT C : IF IOPT = 1 OR 4
A : = 2 OR 5
B : = 3 OR 6
IER : ERROR CODE AND IER > 0, IF AN ERROR OCCURS.
C
4. SUBROUTINES OR FUNCTIONS REQUIRED:
- BBPR - BB01
- BB0B - GWPOB - PHI
- BBAB - GWPAR - QMG - QMACO
- WFUNC3 - GWPOB - PHI
- BBAS - BBAS01 - FLOW

```

```

C
- BBASOB - GWPSOB - FLOW
C
- PHI
C
- BBASAB - GWPSAB - QMG - QMACO
C
- WFUNC4 - WIENER - FLOW
C
- PHI
C*****
DATA XMAX/8.0/, TOL/0.000001/
C
C ***** CHECK ON THE VALIDITY OF PARAMETERS
C
IF ( IOPT.LT.1 .OR. IOPT.GT.6 ) THEN
IER=112
RETURN
ENDIF
GOTO ( 10,20,30,40,50,60 ),IOPT
C
C ***** SET UP APPROPRIATE INITIAL VALUES
C
10 VMAX=XMAX
VMIN=0.
PROB=0.
CALL BBPR( A,B,VMAX,PROB,IER )
GO TO 90
20 VMAX=B
VMIN=0.
CALL BBPR( VMIN,B,C,PROB,IER )
CALL BBPR( B,B,C,PROB,IER )
GO TO 90
30 VMAX=A
VMIN=1.
CALL BBPR( A,VMIN,C,PROB,IER )
CALL BBPR( A,A,C,PROB,IER )
GO TO 90
40 VMAX=XMAX
VMIN=0.
PROB=0.
CALL BBAS( A,B,VMAX,PROB,IER )
GO TO 90
50 VMAX=B
VMIN=0.
CALL BBAS( VMIN,B,C,PROB,IER )
CALL BBAS( B,B,C,PROB,IER )
GO TO 90
60 VMAX=A
VMIN=1.
CALL BBAS( A,VMIN,C,PROB,IER )
CALL BBAS( A,A,C,PROB,IER )
90 IF ( IER.NE.0 ) RETURN
C
IF ( ABS(PROB-P).LT.TOL ) THEN
GOTO ( 310,320,330,310,320,330 ), IOPT
310 C = VMIN
GO TO 390
320 A = VMIN
GO TO 390
330 B = VMIN
GO TO 390
ELSE
IF ( ABS(PROB-P).LT.TOL ) THEN
GOTO ( 360,370,380,360,370,380 ), IOPT
360 C = VMAX
GO TO 390
370 A = VMAX
GO TO 390
380 B = VMAX
GO TO 390
ENDIF
ENDIF
C
IF ( PROB.GT.P.OR.PROB.LT.P ) THEN
IER=132
RETURN
ENDIF
C
C ***** ITERATIVE PROCEDURE STARTS FROM HERE
C
ITER = 1
100 VNEW = VMIN + (VMAX-VMIN)/2.0
WRITE(*,992)ITER
992 FORMAT(' + THE COMPUTATIONS ARE IN PROGRESS, ITERATION NO. =',15)
ITER = ITER + 1
C
GOTO ( 110,120,130,140,150,160 ), IOPT
C
110 CALL BBPR( A,B,VNEW,PROBN,IER )
GO TO 190
120 CALL BBPR( VNEW,B,C,PROBN,IER )
GO TO 190
130 CALL BBPR( A,VNEW,C,PROBN,IER )
GO TO 190
140 CALL BBAS( A,B,VNEW,PROBN,IER )
GO TO 190
150 CALL BBAS( VNEW,B,C,PROBN,IER )
GO TO 190
160 CALL BBAS( A,VNEW,C,PROBN,IER )
190 IF ( IER.NE.0 ) RETURN
C
C ***** IF THE DIFFERENCE IS LESS THAN THE GIVEN TOLEANCE
C THEN STOP THE ITERATIONS
C
IF ( ABS(PROBN-P).GT.TOL ) THEN
IF ( P.GT.PROBN ) THEN
VMIN = VNEW
ELSE

```

```

      VMAX = VNEW
    ENDIF
    GO TO 100
  ELSE
    GOTO ( 210,220,230,210,220,230 ), IOPT
210    C = VNEW
    GO TO 290
220    A = VNEW
    GO TO 290
230    B = VNEW
290    CONTINUE
  ENDIF
390    RETURN
  END

```

```

C=====
C
      SUBROUTINE WPOB( B,C,PROB )
C*****
C + + + + SPECIAL CASE OF WPAB(A,B,C,PROB) WHEN A = 0
C + + + + SPECIAL CASE OF GWPB(B,C,D,PROB) WHEN D = 0
C*****
C
C 1. WPOB - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C     $PROB = P \{ W(T) < C : 0 < T < B \}$ 
C
C 3. PARAMETERS:
C    IN   B   : SEE THE ABOVE FORMULA
C        C   : "
C    OUT  PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - PHI
C
C 5. REFERENCES
C    - CHUNG( 1986, (3.2))
C*****
      BB = SQRT( B )
      PROB = 2.0 * PHI( C/BB ) - 1.0
      RETURN
      END

```

```

C=====
C
      REAL FUNCTION WFUNC1 ( U )
C*****
C
C 1. WFUNC1 - FORTRAN 77 REAL FUNCTION
C
C 2. THIS FUNCTION CALCULATES THE FOLLOWING FORMULA
C
C     $WFUNC1 = \exp(-U^2/2) * [2 * PHI(CC/BB) - 1] / \sqrt{2 * PH}$ 
C    WHERE  $CC = C - \sqrt{A} * U$  AND  $BB = \sqrt{B - A}$ 
C
C 3. PARAMETERS:
C    -INFINITE < U < C/SQRT(A)
C
C 4. SUBROUTINES AND FUNCTIONS REQUIRED:
C    - PHI
C*****
      COMMON /FUNCT/A,B,C,DUM(7)
      DATA PH/3.14159265359/
C
      BB = SQRT( B - A )
      CC = C - SQRT( A ) * U
      PP = 2.0 * PHI( CC/BB ) - 1.0
      CALL FLOW ( UFLOW, OFLOW )
      EUFLOW = -ALOG( UFLOW )
      UUM = U^2/2.0
      IF ( UUM.GT.EUFLOW ) THEN
        UUM = EUFLOW
      ENDIF
      WFUNC1 = PP * EXP( -UUM ) / SQRT(2.0*PH)
      RETURN
      END

```

```

C=====
C
      SUBROUTINE WPAB ( A,B,C,PROB )
C*****
C + + + + SPECIAL CASE OF GWPAB(A,B,C,D,PROB) WHEN D = 0
C*****
C
C 1. WPAB - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C     $PROB = P \{ W(T) < C : A < T < B \}$ 
C
C 3. PARAMETERS:
C    IN   A   : SEE THE ABOVE FORMULA
C        B   : "
C        C   : "
C    OUT  PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - WFUNC1 - PHI
C    - QNG - QMACO
C
C 5. REFERENCES

```

```

C    - CHUNG ( 1986, (3.9))
C    - CSORGO ( 1976, (3.4) OF P. 554)
C    - QUADPACK ( 1983, SUBROUTINE QNG( P.130-136))
C    - RENYI ( 1953, (3.6) OF P. 208)
C*****
      COMMON /FUNCT/A1,B1,C1,DUM(7)
      EXTERNAL WFUNC1
      DATA EPSABS/0.000005/, EPSREL/0.000005/, XMIN/-8.0/
C
      A1 = A
      B1 = B
      C1 = C
      XMAX = C/SQRT(A)
C
C * * * * CALL QNG FOR NUMERICAL INTEGRATIONS WITH FUNCTION WFUNC1
C
      CALL QNG(WFUNC1,XMIN,XMAX,EPSABS,EPSREL,RESULT,ANSE,NEVAL,IER)
      IF ( IER.NE.0 ) THEN
        PROB = 121.
      ELSE
        PROB = RESULT
      ENDIF
      RETURN
      END

```

```

C=====
C
      SUBROUTINE WPASOB ( B,C,PROB )
C*****
C + + + + SPECIAL CASE OF WPASAB(A,B,C,PROB) WHEN A = 0
C + + + + SPECIAL CASE OF WIEN1(C2,C1,B,PROB) WHEN C1 = C2 = C
C*****
C
C 1. WPASOB - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C    PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C     $PROB = P \{ -C < W(T) < C : 0 < T < B \}$ 
C
C 3. PARAMETERS:
C    IN   B   : SEE THE ABOVE FORMULA
C        C   : "
C    OUT  PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C    - FLOW
C
C 5. REFERENCES
C    - CHUNG( 1986, (4.4))
C    - FELLER( 1966, (5.11) OF P. 330)
C*****

```

```

      DATA IMAX/100/, PH/3.14159265359/
C
      CALL FLOW( UFLOW,OFLOW )
      EUFLOW = -ALOG( UFLOW )
      CC = PH * PH * B / ( 8.0 * C * C )
      SIGN = 1.0
      SUM = EXP( -CC )
      DO 10 K = 1, IMAX
        KK = 2 * K + 1
        XK = FLOAT( KK )
        ARG = XK * XK * CC
        IF ( ARG .GT. EUFLOW ) THEN
          GO TO 20
        ELSE
          SIGN = -1.0 * SIGN
          SUM = SUM + SIGN * EXP( -ARG ) / XK
        ENDIF
      10 CONTINUE
      20 PROB = SUM * 4.0 / PH
      RETURN
      END

```

```

C=====
C
      REAL FUNCTION WFUNC2 ( U )
C*****
C
C 1. WFUNC2 - FORTRAN 77 REAL FUNCTION
C
C 2. THIS FUNCTION CALCULATES THE FOLLOWING FORMULA
C
C     $WFUNC2 = \exp(-U^2/2) * PROB[-C-U*AA < W(T) < C-U*AA : 0 < T < B-A] / \sqrt{2 * PH}$ 
C    WHERE  $AA = \sqrt{A}$ 
C
C 3. PARAMETERS:
C    0 < U < C/SQRT(A)
C
C 4. SUBROUTINES AND FUNCTIONS REQUIRED:
C    - WFUNC2 - WIEN1 - PHI
C    - QNG - QMACO
C*****
      COMMON /FUNCT/ A,B,C,DUM(7)
      DATA PH/3.14159265359/
C
      UU = SQRT(A) * U
      BB = B - A
C
C * * * * CALL WIEN1 TO COMPUTE PROB = PROB[-C-UU < W(T) < C-UU : 0 < T < B-A]
C
      CALL WIEN1 ( -C-UU, C-UU, BB, PROB )
      CALL FLOW ( UFLOW, OFLOW )

```

```

EUFLOW = -ALOG( UFLOW )
UUM = U*U/2.0
IF ( UUM.GT.EUFLOW ) THEN
  UUM = EUFLOW
ENDIF
WFUNC2 = EXP ( -UUM ) * PROB / SQRT ( 2.*PH )
RETURN
END

C
C *****
C SUBROUTINE WPASAB ( A,B,C,PROB )
C *****
C + + + + SPECIAL CASE OF GWPSAB(A,B,C,D,PROB) WHEN D = 0
C *****
C
C 1. WPASAB - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILIY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C  $PROB = P \{ -C < W(T) < C; A < T < B \}$ 
C
C 3. PARAMETERS:
C IN A : SEE THE ABOVE FORMULA
C B : "
C C : "
C OUT PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - WFUNC2 - WIEN1 - PHI
C - QNG - QMACO
C
C 5. REFERENCES
C - CHUNG( 1986, (4.11))
C - QUADPACK( 1983, SUBROUTINE QNG( P.130-136))
C *****
COMMON /FUNC/ A1,B1,C1,DUM(7)
EXTERNAL WFUNC2
DATA EPSABS/0.000005/, EPSREL/0.000005/, XMIN/0.0/
C
A1 = A
B1 = B
C1 = C
P = C / SQRT( A )
C
C * * * * CALL QNG FOR NUMERICAL INTEGRATIONS WITH FUNCTION WFUNC1
C
CALL QNG(WFUNC2,XMIN,P,EPSABS,EPSREL,RESULT,ABSE,NEVAL,IER)
IF ( IER.NE.0 ) THEN
  PROB = 122.0
ELSE
  PROB = RESULT * 2.0
ENDIF
RETURN
END

C
C *****
C SUBROUTINE WIEN1 ( C2,C1,B,PROB )
C *****
C + + + + SPECIAL CASE OF WIENER(C2,D2,C1,D1,B,PROB) WHEN D2=D1=0
C *****
C
C 1. WIEN1 - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILIY OF THE WIENER PROCESS OF THE FOLLOWING FORM:
C
C  $PROB = P \{ C2 < W(T) < C1; 0 < T < B \}$ 
C
C IT IS A SPECIAL CASE OF THE ROUTINE WIENER WHEN D2 = D1 = 0
C
C 3. PARAMETERS:
C IN B : SEE THE ABOVE FORMULA
C C2 : " AND C2 < 0
C C1 : " C1 > 0
C OUT PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - PHI
C
C 5. REFERENCES
C - CHUNG( 1986, (4.2) AND (4.3))
C - CSORGO( 1967, (7.5) OF P. 553)
C - FELLER( 1966, (5.8) OF P. 329 AND (5.9) OF P. 330)
C *****
DATA IMAX/100/, TOL/1.0E-9/
C
BB = SQRT( B )
CB1 = C1 / BB
CB2 = C2 / BB
CC = CB1 - CB2
SIGN = 1.0
SUM = 1.0
DO 10 K = 1, IMAX
  CBK = -FLOAT( K ) * CC
  ADD = PHI( CBK + CB1 ) + PHI( CBK - CB2 )
  IF ( ABS(ADD).LT.TOL ) GO TO 20
  SIGN = -1.0 * SIGN
  SUM = SUM + 2.0 * ADD * SIGN
10 CONTINUE
20 PROB = SUM
RETURN

```

```

END
C
C *****
C SUBROUTINE BB01( C,PROB )
C *****
C + + + + SPECIAL CASE OF BB0B(B,C,PROB) WHEN B = 1
C *****
C
C 1. BB01 - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILIY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:
C
C  $PROB = P \{ B(T) < C; 0 < T < 1 \}$ 
C
C 3. PARAMETERS:
C IN C : SEE THE ABOVE FORMULA
C OUT PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C NONE
C *****
PROB = 1.0 - EXP ( -2.0*C*C )
RETURN
END

C
C *****
C SUBROUTINE BB0B ( B,C,PROB )
C *****
C + + + + SPECIAL CASE OF BBAB(A,B,C,PROB) WHEN A = 0
C + + + + SPECIAL CASE OF GWPOB(BB,C,D,PROB) WHEN D=C, BB=B/(1-B)
C *****
C
C 1. BBAS - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILIY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:
C
C  $PROB = P \{ B(T) < C; 0 < T < B \}$ 
C
C 3. PARAMETERS:
C IN B : SEE THE ABOVE FORMULA
C C : "
C OUT PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - GWPOB - PHI
C
C 5. REFERENCES:
C - CHUNG ( 1986, (3.3))
C - HALL AND WELLNER( 1980, (2.8) OF P. 136)
C - KOZLOL AND BYAR( 1975, (2.3) OF P.508)
C *****
BB = B / ( 1. - B )
CALL GWPOB ( BB,C,C,PROB )
RETURN
END

C
C *****
C SUBROUTINE BBAB ( A,B,C,PROB )
C *****
C + + + + SPECIAL CASE OF GWPAB(AA,BB,C,D,PROB) WHEN
C AA = A/(1-A), BB = B/(1-B) AND C = D
C *****
C
C 1. BBAB - FORTRAN 77 SUBROUTINE
C
C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILIY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:
C
C  $PROB = P \{ B(T) < C; A < T < B \}$ 
C
C 3. PARAMETERS:
C IN A : SEE THE ABOVE FORMULA
C B : "
C C : "
C OUT PROB : "
C
C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - GWPAB - QNG - QMACO
C - WFUNC3 - GWPOB - PHI
C
C 5. REFERENCES:
C - CHUNG ( 1986, (3.10))
C - CSAKI ( 1981, (2.17) OF P. 259)
C - CSORGO ( 1983, (4.2.10) OF P.37)
C *****
AA = A / ( 1. - A )
BB = B / ( 1. - B )
CALL GWPAB( AA,BB,C,C,PROB )
RETURN
END

C
C *****
C SUBROUTINE BBAS01( C,PROB )
C *****
C + + + + SPECIAL CASE OF BBASOB(B,C,PROB) WHEN B = 1
C *****
C
C 1. BBAS01 - FORTRAN 77 SUBROUTINE
C

```

C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:

C $PROB = P \{ -C < B(T) < C : 0 < T < 1 \}$

C 3. PARAMETERS:

C IN C : SEE THE ABOVE FORMULA
C OUT PROB :

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - FLOW

```
C *****
C DATA IMAX/100/
C CALL FLOW(UFLOW,OFLOW)
C EUFLOW = -ALOG(UFLOW)
C CC = 2. * C * C
C SIGN = 1.0
C SUM = 0.
C DO 10 K = 1, IMAX
C   UP = CC * K * K
C   IF (UFLOW.GE.EUFLOW) THEN
C     GO TO 20
C   ELSE
C     SIGN = SIGN * (-1)
C     SUM = SUM + SIGN * EXP(-UP)
C   ENDIF
C 10 CONTINUE
C 20 PROB = 1.0 + 2.0 * SUM
C RETURN
C END
```

C SUBROUTINE BBASOB (B,C,PROB)

C *****
C + + + + SPECIAL CASE OF BBASAB(A,B,C,PROB) WHEN A = 0
C + + + + SPECIAL CASE OF GWPSOB(BB,C,D,PROB) WHEN C=D, BB=B/(1-B)
C *****

C 1. BBASOB - FORTRAN 77 SUBROUTINE

C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:

C $PROB = P \{ -C < B(T) < C : 0 < T < B \}$

C 3. PARAMETERS:

C IN B : SEE THE ABOVE FORMULA
C C :
C OUT PROB :

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - GWPSOB - FLOW
C - PHI

C 5. REFERENCES:

C - CHUNG (1986, (4.5))
C - HALL AND WELLNER (1980, (2.9) OF P. 136)

```
C *****
C BB = B / ( 1. - B )
C CALL GWPSOB(BB,C,C,PROB)
C RETURN
C END
```

C SUBROUTINE BBASAB(A,B,C,PROB)

C *****
C + + + + SPECIAL CASE OF GWPSAB(AA,BB,C,D,PROB) WHEN
C AA=A/(1-A), BB=B/(1-B) AND C=D
C *****

C 1. BBASAB - FORTRAN 77 SUBROUTINE

C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE BROWNIAN BRIDGE PROCESS OF THE FOLLOWING FORM:

C $PROB = P \{ -C < B(T) < C : A < T < B \}$

C 3. PARAMETERS:

C IN A : SEE THE ABOVE FORMULA
C B :
C C :
C OUT PROB :

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - GWPSAB - QNG - QMACO
C - WFUNC4 - WIENER - FLOW
C - PHI

C 5. REFERENCES:

C - CHUNG (1986, (4.12))

```
C *****
C AA = A / ( 1. - A )
C BB = B / ( 1. - B )
C CALL GWPSAB( AA,BB,C,C,PROB )
C RETURN
C END
```

C SUBROUTINE GWPOB (B,C,D,PROB)

C *****
C + + + + SPECIAL CASE OF GWPAB(A,B,C,D,PROB) WHEN A = 0

C *****

C 1. GWPOB - FORTRAN 77 SUBROUTINE

C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:

C $PROB = P \{ W(T) < C + D*T : 0 < T < B \}$

C 3. PARAMETERS:

C IN B : SEE THE ABOVE FORMULA
C C :
C D :
C OUT PROB :

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - PHI

C 5. REFERENCES:

C - CHUNG (1986, (3.1))

```
C *****
C BB = SQRT(B)
C C1 = ( C + D*B )/BB
C C2 = (-C + D*B)/BB
C PROB = PHI( C1 ) - EXP( -2.0*C*D ) * PHI( C2 )
C RETURN
C END
```

C REAL FUNCTION WFUNC(U)

C *****

C 1. WFUNC - FORTRAN 77 REAL FUNCTION

C 2. IT CALCULATES THE FOLLOWING FORMULA

C $WFUNC = EXP(-U^2/2) * PROB(W(T) < Q - U*AA + D*T : 0 < T < B - A) / SQRT(2*PH)$
C WHERE $Q = C + D*A$ AND $AA = SQRT(A)$

C 3. PARAMETERS:

C -INFINITE $U < C + D*A$

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:
C - GWPOB - PHI

C 5. REFERENCES:

C - CHUNG (1986, (3.8))

```
C *****
C COMMON /FUNC/A,B,C,D,DUM(6)
C DATA PH/3.14159265359/
C BB = B - A
C CC = C + D*A - SQRT(A)*U
C CALL GWPOB( BB,CC,D,PROB )
C CALL FLOW ( UFLOW, OFLOW )
C EUFLOW = -ALOG( UFLOW )
C UUM = U*U/2.0
C IF ( UUM.GT.EUFLOW ) THEN
C   UUM = EUFLOW
C ENDIF
C WFUNC = EXP( -UUM ) * PROB / SQRT(2.*PH)
C RETURN
C END
```

C SUBROUTINE GWPAB (A,B,C,D,PROB)

C *****

C 1. GWPAB - FORTRAN 77 SUBROUTINE

C 2. THE ROUTINE CALCULATES AN APPROXIMATION RESULT OF THE
C PROBABILITY OF THE WIENER PROCESS OF THE FOLLOWING FORM:

C $PROB = P \{ W(T) < C + D*T : A < T < B \}$

C 3. PARAMETERS:

C IN A : SEE THE ABOVE FORMULA
C B :
C C :
C D :
C OUT PROB :

C 4. SUBROUTINES OR FUNCTIONS REQUIRED:

C - QNG - QMACO
C - WFUNC3 - GWPOB - PHI

C 5. REFERENCES:

C - CHUNG (1986, (3.5))

```
C *****
C COMMON /FUNC/A1,B1,C1,D1,DUM(6)
C EXTERNAL WFUNC3
C DATA EPSABS/0.000005/, EPSREL/0.000005/, XMIN/-8.0/
```

```
C A1 = A
C B1 = B
C C1 = C
C D1 = D
C XMAX = ( C + D*A ) / SQRT(A)
```

C * * * * CALL QNG FOR NUMERICAL INTEGRATION WITH WFUNC3

C CALL QNG(WFUNC3,XMIN,XMAX,EPSABS,EPSREL,RESULT,ABSE,NEVAL,IER)



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